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Vibration analysis of arbitrary straight-sided quadrilateral plates using a simple first-order shear deformation theory

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ABSTRACT

This paper studies the free vibration characteristics of arbitrary straight-sided quadrilateral plates using a simple first-order shear deformation theory (SFSDT), which contains only four unknown displacement components. The arbitrary straight-sided quadrilateral plate is mapped into a unit square plate uniformly, and accordingly, the problem can be solved directly by the existing vibration modeling method of rectangular plate. The admissible functions of displacements are generally expressed as superposition of the periodic functions based on the improved Fourier series method. All the series expansion coefficients can be determined by the Rayleigh-Ritz procedure. Combined with the artificial virtual spring technology, the present method could be used to analyze the vibration characteristics of quadrilateral plates under arbitrary boundary conditions. Convergence and accuracy of the present method are checked out through some numerical examples of plate with rectangular, skew, trapezoidal and general quadrilateral shapes, and various boundary conditions. In addition, some new results and new conclusions have been given as the benchmark for future research.

Introduction

Plates of arbitrary straight-sided quadrilateral configurations are commonly used as the basic structural component for aerospace, submarines, automobile industry and so on. The analysis of its mechanical properties is a key issue for meeting strict dynamic requirements of structural design. Thus, unified mathematical model and reliable solution method to predict the vibration behavior of these plates is of the utmost importance. On this foundation, with the continuous efforts of the researchers over recent decades, there are huge amount of investigations on the vibration of the quadrilateral plate with varying geometry properties and/or boundary conditions. Numerous typical theories of plates have been put forward and developed. These theories can be roughly fall into two categories: three-dimensional (3D) theory and theories on the basis of the reduction of 3D formulation to two-dimensional (2D). The most precise results can be calculated by the 3D theory [1], but at the same time, it also means more complicated solving process and greater computational cost. There are mainly three types of reduction techniques: classical plate theory (CPT) [2–15], Mindlin plate theory (MPT) [16–29] and the higher-order shear deformation theories (HSDTs) [30,31]. The CPT is chiefly applied to thin plates because it disregards the influences of transverse shear deformation. The MPT can be applied to plates of moderately thickness for the reason that the influence of transverse shear deformation is incorporated and the HSDT are suitable for very thick plates.

Once a suitable plate model is formulated, the vibration problems can be solved by means of analytical or numerical methods. Geannakakes *et al.* [3] analyzed the vibration of quadrilateral plates based on the semi-analytical finite strip method, the serendipity function was used to transform the plate to the natural coordinate system, the product of beam orthogonal polynomials and the Hermitian polynomials was chosen as the displacement function. McGee *et al.* [4] assumed that the bending displacement function contains complete algebraic polynomial in mathematics to illustrate the angular function of the singular behavior of the bending stress, the exact value of the natural frequency was obtained using the Ritz method. Gang *et al.* [5] chosen the B3-spline functions as the displacement function to analyze the static and dynamic behavior of arbitrary quadrilateral plates. It is easy to meet boundary conditions by this kind of displacement functions. Differential quadrature is an effective numerical method in engineering practice, which has the advantages of simple mathematical formula, low calculation amount and high precision. Wang *et al.* [6] first applied the differential quadrature method to analyze the free bending vibration characteristics of parallelogram plates with clamped

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boundaries and simply supported boundary conditions. Then, Bert and Malik [7] extended this method to the vibration analysis of clamped and simply supported irregular quadrilateral thin plates by reconstructing the quadrature rule. Shu *et al.* [8] made some improvements based on the method of Ref. [7], they derived the control equation and boundary equation of plates in the curvilinear coordinate system. Liew and Han [17] investigated the vibration features of the quadrilateral Mindlin plate by using a four-node differential quadrature method. Karami and Malekzadeh [9] developed the differential quadrature method to evaluate the weight coefficients of four order partial differential equations with less computation. Then, he extended this method to arbitrary straight-sided quadrilateral functionally graded plates [21], orthotropic nanoplates [22] and laminated plates [23,24]. Xing *et al.* [10] combined the differential quadrature and the finite element method to analyze free vibrations for the thin plate with curvilinear domain. The Non-Uniform Rational B-Splines (NURBS) based isogeometric approach is another effective method to study the free vibration of arbitrary shaped structures. Fantuzzi *et al* [25] used this approach to map arbitrarily shaped structures into the computational domain, the governing equation of each element was solved by the differential quadrature technique. Then, they extended this approach to the free vibrations of arbitrarily shaped laminated shell structures [31,32]. Liu and his students [13,14], Jin and his students [15,33] also used this approach to analyze the in-plane and bending vibration of plates in curvilinear domains. In the same year, Fantuzzi *et al.* [26] studied the free vibration of laminated arbitrary shaped plates by using the strong finite elements, they firstly used three different Fourier expansion-based differential quadrature methods to solve the system equations of the structure element. Bermani and Liew [16] adopted the pb-2 Ritz method to study the free vibration of the quadrilateral thick plate. The product of two-dimensional polynomials and a basic function was used as the admissible functions of displacement. Dozio and Carrera [30] presented a variable kinematic Ritz method which combined the Carrera unified expression and pb-2 Ritz method for the vibration problem of the isotropic arbitrary quadrilateral thin and thick plate. Huang *et al.* [19] applied the Ritz method to study the free vibration of cantilevered skew, triangular and trapezoidal thick plates. Quintana and Nallim [20] used non-orthogonal triangular coordinates and Ritz methods to investigate the free vibration characteristics of the polygonal plate with variable thickness. The discrete singular convolution method, which is a numerical method for integral equations solution, was adopted to the free vibration analysis of arbitrarily-shaped quadrilateral plates by Civalek [11]. Zhang *et al* [27–29] proposed an improved moving least-squares Ritz (IMLS-Ritz) method to study the mechanical behavior of quadrilateral composite plates based on the FSDT. Recently, Guan *et al.* [12] proposed a Fourier expansion solution for investigating the bending vibration characteristic of the sector like thin plate, which includes arbitrary quadrilateral plates and triangular plates.

Despite the complicacy of the solution procedure had been reduced by these 2D theories, some simplified theories are proposed by researchers for further simplify some special cases. Thai and his colleagues [34–37] applied a SFSDT to investigate the vibration characteristics of the functionally graded plate and the laminated plate. Sadoune *et al.* [38] solved the vibration of the simply supported laminated plate by using an innovative SFSDT. Nevertheless, the simplified theory mentioned above is restricted to the simply supported boundary condition and cannot solve the vibration characteristics for plates with more complicated geometry properties. Accordingly, a simplified theory that can be applied to plates with arbitrary shape and arbitrary boundary conditions should be developed to compensate for the restrictions of existing theories.

In this article, we proposed a unified and accurate approach on the basis of a SFSDT to analyze the free vibration characteristics of arbitrarily-shaped straight-sided quadrilateral plates. The new theory is in conjunction with the improved Fourier series method [39–43] and the artificial boundary spring technique [44–50] which can simulate arbitrary boundary restraints on the boundaries of plates. A four-node coordinate mapping procedure is introduced into the present improved Fourier series method to transform arbitrary straight-sided quadrilateral regions to the unit square domain, such kind method has never been proposed in the existing literature. The admissible displacement functions for quadrilateral plates are uniformly expressed as the standard two-dimensional Fourier series and supplementary series expansion which could avoid the jumping or discontinuities of the boundaries. The Rayleigh-Ritz method is adopted to determine the unknown Fourier series expansion coefficients. Under the current framework, several results for vibration characteristics of the isotropic quadrilateral plate with varying shapes and various elastic boundary restraints are presented, which have shown the excellent convergence, high accuracy and stability by contrast to the result obtained by the corresponding publications and the finite element solution.

Theoretical formulations

Description of the plate geometry

Fig. 1 displays the global coordinate system and geometry model for a straight-sided quadrilateral isotropic plate with uniform thickness *h*. It should be pointed out that the mid-surface displacements for the quadrilateral plate in the *x*, *y* and *z* direction are respectively represented by the symbols *u*, *v* and *w*. In this scheme, the artificial virtual spring technology is introduced to simulate boundary restraints of plates. The spring combination is composed of four groups of translational springs (, k_v, k_w and k_s) and one group of rotational spring (K_w). By varying the spring stiffness to some certain values, simulation of various boundary conditions including classical boundaries and elastic boundaries can be achieved.

In the current study, the authors' main purpose is to develop a novel method for the free vibration of arbitrarily-shaped straight-sided quadrilateral plates with arbitrary boundary conditions. Unlike other methods, the method in current study does not need to mesh the plate structure, so it will not produce discrete error. The arbitrary quadrilateral plate (Fig. 2a) in the *x*-*y* physical domain is transformed into a unite square plate (Fig. 2b) in the *ξ*-*η* coordinate system by means of a special four-node coordinate transformation.

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \sum_{i=1}^{4} N_i(\xi,\eta) \begin{Bmatrix} x_i \\ y_i \end{Bmatrix} \tag{1}$$

in which x_i and y_i denote the coordinates for node *i* of quadrilateral plates. $N_i(\xi,\eta)$ represents the shape functions for the transformation related with node *i*, and it can be defined by:

$$N_i = (-1)^{i+1}(1-\xi_i-\xi)(1-\eta_i-\eta)\,\text{for } i = 1, 2, 3, 4 \tag{2}$$

According to this coordinate transformation and the chain

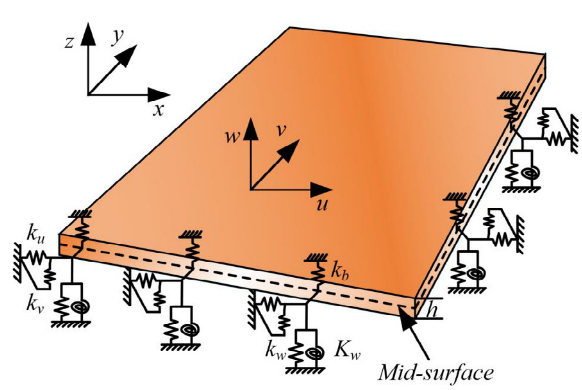

Fig. 1. Coordinate system and geometry model of the quadrilateral plate.