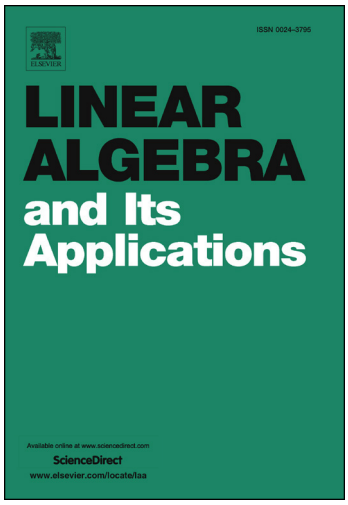

Completely Positive Cones: Are They Facially Exposed?

Qinghong Zhang

PII: S0024-3795(18)30411-7
DOI: https://doi.org/10.1016/j.laa.2018.08.028
Reference: LAA 14707

To appear in: *Linear Algebra and its Applications*

Received date: 24 March 2018
Accepted date: 21 August 2018

Please cite this article in press as: Q. Zhang, Completely Positive Cones: Are They Facially Exposed?, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.08.028

Completely Positive Cones: Are They Facially Exposed?

Qinghong Zhang

Department of Mathematics and Computer Science, Northern Michigan University, Marquette, MI 49855, USA. E-mail address: qzhang@nmu.edu

Abstract

Whether the $n \times n$ completely positive cone for $n \geq 5$ is facially exposed is an open problem raised by Berman, *et. al.* in 2015. In this paper, we give a class of non-exposed faces of the 5×5 completely positive cone, which implies that the 5×5 completely positive cone is not facially exposed. By embedding the 5×5 completely positive cone into the $n \times n$ completely positive cone for $n > 5$, we further show that the $n \times n$ completely positive cone is not facially exposed for $n > 5$.

Keywords: Completely Positive Cones, Copositive Cones, Faces, Facially Exposed Cones

2010 MSC: 15B48, 15A23, 15B99, 52A20

1. Introduction

The copositive cones and their dual cones, namely the completely positive cones, have many applications, and have been topics of research for many years (see [1] [2]). However, due to their complicated structures, knowledge about the geometric aspects (especially facial structures) of the copositive cones and completely positive cones are very limited. In [3], a way of representing all the maximal faces of the copositive cones along with a simple equation for the dimension of each one was given because of the known representations of exposed rays of the completely positive cones. Also in [3] some maximal faces (not all maximal faces) of the completely positive cones and their dimensions were