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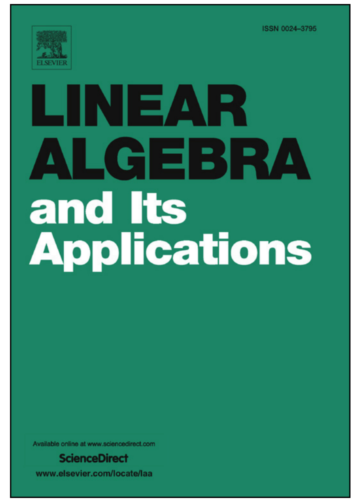

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Spectral Radius of $\{0,1\}$-Tensor with Prescribed Number of Ones

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Abstract

For any r-order $\{0,1\}$-tensor A with e ones, we prove that the spectral radius of A is at most $e^{\frac{r-1}{r}}$ with the equality holds if and only if $e = k^r$ for some integer k and all ones forms a principal sub-tensor $\mathbf{1}_{k\times\cdots\times k}$. We also prove a stability result for general tensor A with e ones where $e = k^r + l$ with relatively small l. Using the stability result, we completely characterized the tensors achieving the maximum spectral radius among all r-order $\{0,1\}$-tensor A with $k^r + l$ ones, for $-r-1 \leq l \leq r$, and k sufficiently large.

MSC: 05C50; 05C35
keywords: Spectral radius, nonnegative tensor, $0,1$-tensor, maximum tensor

1 Introduction

For a real nonnegative square matrix A the spectral radius $\rho(A)$ is the largest eigenvalue of A in modulus, which is real as guaranteed by the Perron-Frobenius theorem. The problem of finding the maximal spectral radius for all $\{0,1\}$-matrices with prescribed number of ones was introduced by Brualdi and Hoffman [1] in 1985. Let $g(e)$ be the maximal spectral radius of A among all $\{0,1\}$-matrices A with e ones. They proved that for each positive integer k, $g(k^2) = g(k^2+1) = k$. When $e = k^2$, the equality holds if A is essentially a $k \times k$ all-1-matrix (inserted by possibly extra rows/columns of 0's). When $e = k^2 + 1$ and $k \geq 3$, the equality is attained for only when a useless additional 1 is put at any place else to a $k \times k$ all-1-matrix. (But for $k = 1$, or 2, there is another A with $\rho(A) = k$.) Friedland [5] solved another cases when $e = k^2 - 1$, $e = k^2 - 4$, or $e = k^2 + l$ for a fixed l and k sufficiently large. In all cases, the matrices with maximum spectral radius are characterized.

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