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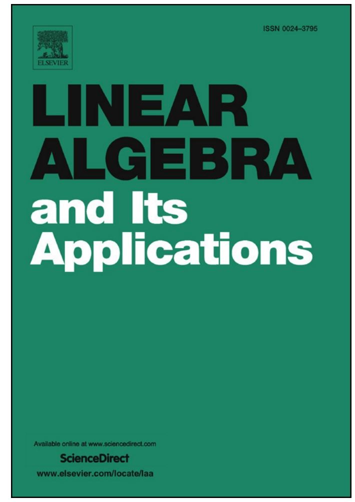

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ON THE CHARACTERISTIC POLYNOMIALS OF MULTIPARAMETER PENCILS

ZHIGUANG HU AND RONGWEI YANG*

ABSTRACT. In this paper we study the characteristic polynomial of multiparameter pencil $z_1A_1 + z_2A_2 + \cdots + z_sA_s$. The main theorem states that a unitary representation of a finitely generated group contains a one-dimensional representation if and only if the characteristic polynomial of its generators contains a linear factor. It follows that a two or three dimensional unitary representation of a finitely generated group is irreducible if and only if the characteristic polynomial of the pencil of its generators is irreducible. The result is of kin to the Dedekind and Frobenius theorem on finite group determinant.

Mathematics Subject Classification (2000): 15A15, 15A54, 20C15.
Key words: characteristic polynomial, multiparameter pencil, normal matrix, group representation, group determinant.

1. Introduction

Let $A = (A_1, \ldots, A_s)$ be a tuple of $n \times n$ complex matrices and I be the identity matrix of the same size. This paper studies the multivariable homogeneous characteristic polynomial

$$Q_A(z) := \det(z_0I + z_1A_1 + z_2A_2 + \cdots + z_sA_s)$$

of the multiparameter pencil $z_1A_1 + z_2A_2 + \cdots + z_sA_s$ which is a natural generalization of the classical characteristic polynomial $\det(A - zI)$ for a single square matrix A. Study of such polynomial began at least as early as the late 19th century when Dedekind and Frobenius started to investigate the so-called group determinant for finite groups (cf. Section 3). It is a natural question which homogeneous polynomial is of the form kQ_A for some scalar k. For research along this line we refer readers to [6, 15, 19] and the references therein. Some other work on the notion of characteristic polynomial of several matrices can be found in [1, 10, 12].

The work here is motivated by the notion of projective spectrum defined by the second author in [20]. For more information about research on projective spectrum we refer readers to [2, 11, 13, 14, 16, 17, 18]. One remarkable application of the multivariable characteristic polynomial is recently made in [2] where, as a consequence of a more general theorem, it is shown that if $A_1, \ldots, A_s$ are normal matrices then they pairwise commute if and ony if their charateristic polynomial $Q_A(z)$ is a product of linear factors. This paper will first give an elementary proof to this fact. The main concern of this paper is regarding characteristic polynomial for finite groups. To be more precise, consider a finitely generated group G with a generating set $S = \{g_1,\ g_2,\ \ldots,\ g_s\}$ and a unitary representation π of G on $\mathbb{C}^n$.

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