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POLYNOMIAL APPROXIMATION IN CONVEX DOMAINS

YU. BRUDNYI

ABSTRACT. We study the phenomenon of strengthening rate of polynomial approximation near boundary points of a convex domain. The main result of the paper presents this type of approximation near conic boundary points of these domains while Gopengauz's counter-example implies impossibility of such approximation near other points of the boundary.

1. FORMULATION OF THE MAIN RESULT

A boundary point x of a convex domain $G \subset \mathbb{R}^d$ is said to be *conic* if there are d linearly independent supporting hyperplanes of G through x .

For instance, the set of conic points of a convex polytope is its vertex set.

Notations. Throughout the paper G stands for a compact convex domain in \mathbb{R}^d and $\Gamma \subset \partial G$ denotes the set of its conic points.

\mathcal{P}_n is the space of polynomials in $x \in \mathbb{R}^d$ of (total) degree n , i.e., the linear hull of the set $\{x^\alpha\}_{|\alpha| \leq n}$, where as usual

$$x^\alpha := \prod_{i=1}^d x_i^{\alpha_i}, \quad |\alpha| = \sum_{i=1}^d \alpha_i, \quad \alpha \in \mathbb{Z}_+^d.$$

The space \mathbb{R}^d is assumed to be equipped by the Euclidean norm

$$|x| := \left(\sum_{i=1}^d x_i^2 \right)^{1/2}, \quad x \in \mathbb{R}^d,$$

and all distances are measured by this norm.

Finally, a function $(f, t) \mapsto \omega_k(f; t)$, $(f, t) \in C(G) \times \mathbb{R}_+$ is given by

$$(1.1) \quad \omega_k(f; t) := \sup_{|h| \leq t} \sup_{G_{kh}} |\Delta_h^k f|(x),$$

where as usual

$$\Delta_h^k f(x) := \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} f(x + jh), \quad h \in \mathbb{R}^d,$$

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Key words and phrases. Polynomial approximation; Convex domain; conic point; Modulus of continuity; Polynomial operator.

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