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POLYNOMIAL APPROXIMATION IN CONVEX DOMAINS

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ABSTRACT. We study the phenomenon of strengthening rate of polynomial approximation near boundary points of a convex domain. The main result of the paper presents this type of approximation near conic boundary points of these domains while Gopengauz's counter-example implies impossibility of such approximation near other points of the boundary.

1. Formulation of the main result

A boundary point x of a convex domain $G \subset \mathbb{R}^d$ is said to be *conic* if there are d linearly independent supporting hyperplanes of G through x.

For instance, the set of conic points of a convex polytope is its vertex set.

Notations. Throughout the paper G stands for a compact convex domain in $\mathbb{R}^d$ and $\Gamma \subset \partial G$ denotes the set of its conic points.
$\mathcal{P}_n$ is the space of polynomials in $x \in \mathbb{R}^d$ of (total) degree n, i.e., the linear hull of the set $\{x^\alpha\}_{|\alpha|\le n}$, where as usual

$$x^\alpha := \prod_{i=1}^{d} x_i^{\alpha_i}, \qquad |\alpha| = \sum_{i=1}^{d} \alpha_i, \qquad \alpha \in \mathbb{Z}_+^d.$$

The space $\mathbb{R}^d$ is assumed to be equipped by the Euclidean norm

$$|x| := \Big(\sum_{i=1}^{d} x_i^2\Big)^{1/2}, \qquad x \in \mathbb{R}^d,$$

and all distances are measured by this norm.

Finally, a function $(f,t) \mapsto \omega_k(f;t)$, $(f,t) \in C(G) \times \mathbb{R}_+$ is given by

$$\omega_k(f;t) := \sup_{|h|\le t} \sup_{G_{kh}} \big|\Delta_h^k f\big|(x), \tag{1.1}$$

where as usual

$$\Delta_h^k f(x) := \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} f(x+jh), \quad h \in \mathbb{R}^d,$$

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