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ACCURATE SOLUTIONS OF PRODUCT LINEAR SYSTEMS ASSOCIATED WITH RANK-STRUCTURED MATRICES

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Abstract. In this paper, we consider how to accurately solve linear systems associated with a wide class of rank-structured matrices containing the well-known Vandermonde and Cauchy matrices, i.e., consecutive-rank-descending (CRD) matrices. We provide a mechanism to guarantee that the inverse of any product of CRD matrices is generated in a subtraction-free manner. With the mechanism, the solutions of linear systems associated with such products are accurately determined by the parameters of CRD factors, and we then accurately compute the solutions as warranted by these parameters. In particular, linear systems associated with products of Vandermonde and Cauchy matrices, whose nodes satisfy certain positive or negative properties, are solved to high relative accuracy. Error analysis and numerical experiments are provided to confirm the high accuracy.

Key words. linear system, rank-structured matrix, matrix product, parametrization, high relative accuracy

AMS subject classifications. 65F15, 15A18

1. Introduction. When a linear system $Ax = b$ is numerically solved in an "ideal" situation, i.e., the roundoff errors occur only when storing the right-hand vector b on the machine, the computed solution $\hat{x}$ is nicely bounded as

$$|\hat{x} - x| \leq \mu |A^{-1}||b|, \tag{1.1}$$

and if $|A^{-1}||b| = |A^{-1}b|$, then a remarkably high relative accuracy is produced for the computed solution $\hat{x}$ as

$$|\hat{x} - x| \leq \mu |x|. \tag{1.2}$$

Throughout the paper, μ is the unit roundoff, the absolute value of a vector or a matrix is meant componentwise. It is natural to ask whether these "ideal" bounds (1.1) and (1.2) can be pursued for practical linear system solvers. Indeed, these bounds have been derived when the coefficient matrix A is a Vandermonde matrix whose nodes are positive in an increasing order [3], and the key fact is that A can be decomposed as a product of nonnegative bidiagonal matrices B_i $(1 \leq i \leq m)$, i.e., $A = B_1 \ldots B_m$. In this case, the solution $x = B_m^{-1} \ldots B_1^{-1} b$ with $|A^{-1}| = |B_m^{-1} \ldots B_1^{-1}| = |B_m^{-1}| \ldots |B_1^{-1}|$. Taking into account the high relative accuracy for inverses of these bidiagonal matrices, the computed solution $\hat{x}$ satisfies that

$$|\hat{x} - x| \leq O(\mu)|B_m^{-1}| \ldots |B_1^{-1}||b| = O(\mu)|A^{-1}||b|, \tag{1.3}$$

and if $b = (b_i) \in \mathbb{R}^{n \times 1}$ is sign-interchanging, i.e., $(-1)^i b_i \geq 0$, then $|B_m^{-1}| \ldots |B_1^{-1}||b| = |B_m^{-1} \ldots B_1^{-1} b|$ such that the bound is reduced as

$$|\hat{x} - x| \leq O(\mu)|x|.$$

Before proceeding, we remark that nonsingular bidiagonal matrices belong to the class of rank-structured matrices defined as follows:

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