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DSF chain for biorthogonal polynomials and its application to matrix eigenvalue problems

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Abstract

A discrete-time integrable system, called the DSF chain, is derived from the compatibility condition of spectral equations for biorthogonal polynomials associated the discrete Schur flow. The DSF algorithm for computing eigenvalues of a type of matrix eigenvalue problems is designed. A discrete analogue of the Miura transformation between the discrete Schur flow and the qd-algorithm is constructed. Two special solutions to the DSF chain related to q-special functions is presented.

Keywords: Schur flow, qd-algorithm, matrix eigenvalue problems, numerical algorithms, integrable systems, biorthogonal polynomials
2000 MSC: 33D45, 35Q55, 37K10, 37K35, 41A21, 42C05, 65F15

1. Introduction

The Schur flow, introduced by Ammar and Gragg [1], is an integrable system arising from a matrix flow on unitary Hessenberg matrices. The Toda equation is an integrable matrix flow on Jacobi matrices [2] and thus the Schur flow is considered as an analogue of the Toda equation. This class of integrable systems is closely related to the theory of orthogonal polynomials; the Toda equation describes a spectral transformation of orthogonal polynomials [3], while the Schur flow does that of biorthogonal polynomials. In context of physics, the Schur flow is a spatially discrete version of the complex modified KdV equation and equivalent to an integrable system proposed by Ablowitz and Ladik [4] through a variable transformation. As one of the most typical models in integrable systems, the Schur flow appears in many problems in applied mathematics and physics, and has been studied from various points of view.

It is known that some class of integrable systems can be applied to numerical analysis through their time discretizations. A typical example is given by the Toda equation, whose discrete-time analogue is equivalent to the qd-algorithm [5, 6]. The Schur flow also has a discrete-time analogue, which is called the discrete Schur flow. In [7, 8] the authors studied the discrete Schur flow and clarified its basic properties including exact determinant solutions and their long-time asymptotics. Furthermore, they applied the results to numerical analysis and proposed numerical algorithms for solving algebraic equations and calculating Padé approximations.

In this article, we study the underlying relationship between biorthogonal polynomials and the discrete Schur flow, and apply the result to design a numerical algorithm for solving matrix eigenvalue problems. Since the discrete Schur flow was introduced as a discrete spectral transformation

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