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General four-step discrete-time zeroing and derivative dynamics applied to time-varying nonlinear optimization

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ABSTRACT

Time-varying nonlinear optimization (TVNO) problems are considered as important issues in various scientific disciplines and industrial applications. In this paper, the continuous-time derivative dynamics (CTDD) model is developed for obtaining the real-time solutions of TVNO problems. Furthermore, aiming to remedy the weaknesses of CTDD model, a continuous-time zeroing dynamics (CTZD) model is presented and investigated. For potential digital hardware realization, by using bilinear transform, a general four-step Zhang et al discretization (ZeaD) formula is proposed and applied to the discretization of both CTDD and CTZD models. A general four-step discrete-time derivative dynamics (general four-step DTDD) model and a general four-step discrete-time zeroing dynamics (general four-step DTZD) model are proposed on the basis of this general four-step ZeaD formula. Further theoretical analyses indicate that the general four-step DTZD model is zero-stable, consistent and convergent with the truncation error of $\mathbf{O}(g^4)$, which denotes a vector with every entries being $O(g^4)$ with g denoting the sampling period. Theoretical analyses also indicate that the maximal steady-state residual error (MSSRE) has an $O(g^4)$ pattern confirmedly. The efficacy and accuracy of the general four-step DTDD and DTZD models are further illustrated by numerical examples.

1. Introduction

In numerous scientific disciplines and industrial applications, time-varying optimization problems have been widely encountered, and such problems are fundamental and essential [1–10]. Since nonlinear optimization is an important subtopic of optimization problems, researches on this are abundant and many of which have been applied in the engineering fields such as system control [11–13] and signal processing [14–17]. Various algorithms have been proposed and analyzed for nonlinear optimization problems solving [10–27]. For example, in [11], a decomposition scheme is proposed to solve parametric non-convex programs. Such a scheme consists of a fixed number of proximal linearized alternating minimizations and a dual update per time step. The proposed approach is attractive in a real-time distributed context and the performance of the optimality-tracking scheme can be enhanced via a continuation technique. In [13], a self-triggered algorithm is presented to solve a class of convex optimization problems with time-varying objective functions. The algorithm predicts the temporal evolution of the gradient by using known upper bounds on higher order derivatives of the objective function. The method guarantees convergence to arbitrarily small neighborhood of the optimal trajectory in finite time and without incurring Zeno behavior. In [17], based on prediction and correction steps, Simonetto et al. proposed two algorithms with a discrete

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time-sampling scheme. Considering the correction step consists either of one or multiple gradient steps or Newton steps, the two algorithms are termed gradient trajectory tracking (GTT) and Newton trajectory tracking (NTT) algorithms. The two algorithms behave as $O(g^2)$ and in some cases as $O(g^4)$ with g denotes the sampling period. Furthermore, in [16], online algorithms are developed to track solutions of time-varying constrained optimization problems. Resembling workhorse Kalman filtering-based approaches for dynamical systems, the proposed methods involve prediction–correction steps to provably track the trajectory of the optimal solutions of time-varying convex problems and improve the convergence speed of existing prediction–correction methods when applied to unconstrained problems. However, most of these algorithms and methods are dedicated in solving static nonlinear optimization problems, which means that they may not possess the capability of handling time-varying nonlinear optimization (TVNO) problems [20–22]. The main difference between TVNO problems and static nonlinear optimization problems is evidently that TVNO problems change with time. This difference makes the time derivative play an important role in obtaining accurate real-time solutions for TVNO problems.

Traditional numerical algorithms such as the Newton–Raphson iteration (NRI) and other methods [10,18–20] are designed intrinsically for static optimization. Generally speaking, these methods are under the assumption that the optimization problems do not change during the computational time. Thus, the calculated solutions are directly used in the optimization problems after the calculation. Comparing to the traditional numerical algorithms, the neural dynamical approach is superior because of the potential real-time application advantages such as self-adaption, parallel processing, distributed storage and hardware applications [28,29]. In this paper, the derivative dynamics approach is exploited as an intuitive one while the zeroing dynamics approach is exploited for potential predictive power [30–32]. More specifically speaking, a continuous-time derivative dynamics (CTDD) model and a continuous-time zeroing dynamics (CTZD) model are generalized, developed and investigated for TVNO problems.

For potential digital hardware realization, it is necessary to discretize the continuous-time dynamics models. Fortunately, a new class of finite-difference methods and formulas termed Zhang et al. discretization (ZeaD) has been proposed, named and applied by Zhang et al. since 2014 after nearly 8-year search and preparation [33]. It is worth pointing out that ZeaD formulas can be used for stable, convergent and accurate discretization of neural dynamics (i.e., derivative dynamics and zeroing dynamics) and other continuous-time ordinary differential equation (ODE) systems [33].

The rest of this paper consists of five sections. Section 2 presents the general formulation of nonlinear optimization. Then the CTDD model is developed. Since the performance of CTDD model is related to the initial state and perturbations, the CTDD model may not solve TVNO problems in a robust manner. Aiming to remedy this weakness, a CTZD model is then developed. In Section 3, for potential digital hardware realization, a general four-step ZeaD formula is proposed for the discretization of the CTDD and CTZD models. The design procedure is also presented in Section 3. The design procedure guarantees that the general four-step ZeaD formula is zero-stable, consistent and convergent with the truncation error of $O(g^3)$. Based on the general four-step ZeaD formula, the CTDD and CTZD models, the general four-step discrete-time derivative dynamics (general four-step DTDD) model and discrete-time zeroing dynamics (general four-step DTZD) model are proposed, respectively. Then in Section 4, further theoretical analyses are conducted, and the characteristics of the proposed general four-step DTZD model are investigated. We further find out that the general four-step DTZD model has a truncation error of $\mathbf{O}(g^4)$, which denotes a vector with every entries being $O(g^4)$, and that its maximal steady-state residual error (MSSRE) also has an $O(g^4)$ pattern. Furthermore, numerical results of TVNO examples illustrate the efficacy and accuracy of the proposed DTDD and DTZD models in Section 5. Finally in Section 6, the conclusion along with the final remarks of this paper is presented. Before proceeding, a summarization of the main contributions of this paper is worth listing out.

(1) For the first time, by using bilinear transform, a general four-step ZeaD formula having $O(g^3)$ truncation error is proposed and investigated in this paper. Such a general four-step formula can be used for the stable, convergent and accurate discretization of neural dynamics and other continuous-time ODE systems.
(2) By adopting the general four-step ZeaD formula to discretize CTDD and CTZD models, the general four-step DTDD and DTZD models are proposed in this paper. Both general four-step discrete-time dynamics models solve TVNO problems successfully. Note that the general four-step DTDD model is sensitive to the initial state and shows a residual error of $O(g^2)$, and the general four-step DTZD model is robust and has a residual error of $O(g^4)$.
(3) The stability and convergence results of the general four-step DTZD model for TVNO problems solving are proved, with the MSSRE of the model having an $O(g^4)$ pattern.
(4) Numerical experimental results synthesized by the general four-step DTDD and DTZD models are presented, which further substantiate the efficacy and accuracy of the general four-step DTDD and DTZD models. These results collaborate well with the theoretical analyses.

2. Problem formulation, continuous-time models and general ZeaD formula

In this section, the problem formulation of TVNO is presented for further investigation. Based on this problem formulation, the CTDD model and CTZD model are developed. Then, the general ZeaD formula is presented to lay the foundation of the general four-step ZeaD formula.