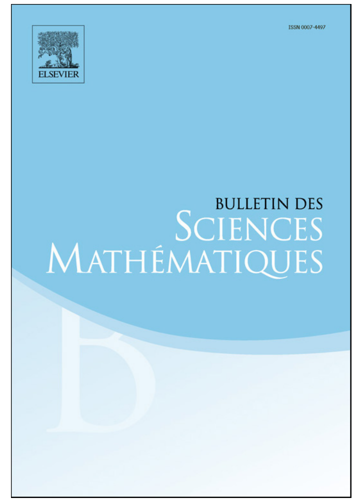

Invariant measures for the non-periodic two-dimensional Euler equation

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INVARIANT MEASURES FOR THE NON-PERIODIC TWO-DIMENSIONAL EULER EQUATION

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Abstract. We construct Gaussian invariant measures for the 2D Euler equation on the plane. We obtain them as the weak limit of those previously considered in [1] for the torus. We show the existence of solution with initial conditions on the support of the measures. Uniqueness and continuity of the velocity flow are proved.

Contents

1. Introduction

Euler equation describes the time evolution of an incompressible non-viscous fluid with constant density. This fundamental equation has been and still is intensively studied. Among the numerous references on the Euler equation, we cite the books [6, 15, 17]. It is known that solutions do not blow up starting from smooth data with finite kinetic energy (T. Kato (1967) [14], C. Bardos (1972) [7] among others). In two dimensions, for bounded domains and when the initial vorticity is bounded, existence, uniqueness and global regularity of solutions was shown (V.I. Yudovich, 1963 [13]); these results were extended, in the framework of weak solutions, to the case where the initial vorticity belongs to L^p, with $p > 1$ and even for $p = 1$, when the vorticity is some finite measure.

A more geometric approach, identifying the solutions of the Euler equation with velocities of geodesics in a space of diffeomorphisms of the underlying state space, was initiated by V. Arnold (1966) [5]. It allowed to show existence of local solutions in some Sobolev spaces (D. G. Ebin and J. Marsden, 1970 [12]).

Much less is known about irregular solutions of the Euler equation. This paper is devoted to a class of such solutions.

In statistical approaches to hydrodynamics, discussed in the physics literature on turbulence, one considers the evolution of probability densities instead of pointwise solutions. A major subject of interest is the search for invariant measures. In particular such measures are important because they can be used to prove the existence and study properties of Euler flows defined almost-everywhere with respect to them.