International Journal of Heat and Mass Transfer 128 (2019) 279–289

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Heat transfer of laminar pulsating flow in a rectangular channel

R. Blythman [a,*], T. Persoons [a], N. Jeffers [b], D.B. Murray [a]

[a] Department of Mechanical and Manufacturing Engineering, Trinity College Dublin, Dublin 2, Ireland
[b] Thermal Management Research Group, Efficient Energy Transfer (ηET) Department, Nokia Bell Labs, Blanchardstown Business & Technology Park, Snugborough Rd, Dublin 15, Ireland

ARTICLE INFO

Article history:
Received 5 April 2018
Received in revised form 11 July 2018
Accepted 23 August 2018

ABSTRACT

Pulsating flow has been found to both enhance and reduce heat transfer, although the explicit conditions that deliver heat transfer enhancement have yet to be identified. The current work builds on experimental and theoretical hydrodynamic analyses of earlier studies to investigate the effect of flow rate pulsations on the driving temperature difference for the case of constant heat flux, using a novel analytical solution to the energy equation in a rectangular channel. It is found that the oscillating temperature profiles are formed primarily as a result of fluid displacement against the temperature gradient, although diffusion at low frequencies and low Prandtl numbers obscures the mechanism. The temperature gradient at the wall is fixed for the case of constant heat flux, despite enhanced velocity gradients at the wall. The overall time-averaged change in Nusselt number with respect to steady flow is universally negative due to an enhanced axial heat flow towards the channel entrance, in agreement with similar studies in pipes. However, analysis of the time-dependent bulk temperature predicts that heat transfer is enhanced over the second half-cycle, where the flow rate is below it's mean value. Hence, future work should investigate heat transfer enhancement using truncated sinusoidal flow rates.

1. Introduction

1.1. Heat transfer enhancement in pulsating flow

Unsteady flows are often exploited to disrupt the hydrodynamic and thermal boundary layers, and overall heat transfer enhancements have been measured for flows with complex boundary conditions, such as pulsating and synthetic jets [1,2], pulsations in short minichannel heat sinks at low Reynolds numbers [3] and oscillatory flow in baffled tubes [4]. However, an understanding of the explicit conditions that deliver heat transfer enhancement has yet to be achieved. Since pulsating flow is of primary interest, heat transfer enhancement is defined relative to a steady flow with a flow rate equal to the time-average of the pulsating flow rate.

In general, a dichotomy exists between hydrodynamic and heat transfer experiments measuring pulsating flows. While the velocity field has been measured on a local time-dependent basis, theoretical and experimental studies have typically chased time- and space-averaged heat transfer enhancement. While a time-average increase in heat transfer is of course the ultimate goal, this macroscopic treatment has failed to establish consensus on the potential of flow unsteadiness to enhance heat transfer, by obscuring the time-dependent behaviour of physical mechanisms. For example, the discrepancy between heat transfer enhancement through changes in the bulk temperature and by changes in the temperature gradient at the wall has not been highlighted in the literature, to the best of the authors' knowledge. Specifically, theoretical studies find negligible or slight alterations in time-averaged heat transfer for sinusoidal or pseudo-sinusoidal pulsations under constant heat flux [5,6], while less restrictive boundary conditions identify intervals of instantaneous heat transfer enhancement [7,8]. Since the heat transfer coefficient describes the ratio of the heat flux at the wall to the driving temperature difference, any change with pulsation encompasses the overall net effect of changes in the numerator and denominator. Hence, the decomposition of any heat transfer alteration into changes in heat flux and temperature difference presents more opportunity to exploit the underlying mechanisms. In the current work, the effect of flow rate pulsations on the driving temperature difference is investigated for the case of constant heat flux, using a novel solution to the energy equation in a rectangular channel. In a future work, the behaviour of the heat flux at the wall of a pulsating flow will be analysed using the constant temperature boundary condition.

1.2. Axial diffusivity mechanism

Much of the initial progress was made in the analogous problem of mass transfer with a non-porous wall. Taylor [9] and Aris [10] determined that steady fluid flow enhances the spreading of a

* Corresponding author.
E-mail address: blythmar@tcd.ie (R. Blythman).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.08.109
0017-9310/

Nomenclature

$\hat{a}, \hat{b}$	channel width, height [m]
A_0	dimensionless amplitude (=$2\langle\chi_A\rangle/D_h$)
c	heated fraction of cross-sectional perimeter
c_p	specific heat capacity [J/(kg·K)]
dNu	heat transfer enhancement
D_h	hydraulic diameter [m]
f	oscillation frequency [Hz]
h	heat transfer coefficient [W/(m^2·K)]
i	imaginary unit (=$\sqrt{-1}$)
j, k	summation indices
$\Im$	imaginary part of complex number
k	thermal conductivity [W/(m·K)]
$\widehat{L}$	heated length [m]
m, n	summation indices
$\hat{p}$	pressure [Pa]
Pe	Péclet number (=$Re{\cdot}Pr$)
$\hat{q}$	heat flux [W/m^2]
Nu	Nusselt number (=$1/(\theta_w - \theta_b)$)
Pr	Prandtl number (=ν/α_f)
$\widehat{Q}$	flow rate [m^3/s]
$\Re$	real part of complex number
Re	Reynolds number (=$\langle\hat{u}\rangle D_h/\nu$)
$\hat{t}$	time [s]
T	temperature [K]
$\hat{u}$	velocity in the axial direction [m/s]
Wo	Womersley number (=$\frac{1}{2}D_h\sqrt{\omega/\nu}$)
$\hat{x}$	axial flow coordinate [m]
$\hat{y}, \hat{z}$	coordinates normal to flow direction [m]

Greek symbols

α	thermal diffusivity [m^2/s]
β	functions defined by Eqs. (6b) and (12)
$\hat{\delta}_\nu$	Stokes layer thickness [m]
θ	temperature
λ	dimensionless oscillation period (=$2\pi/Wo^2$)
μ	viscosity [kg/(m·s)]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
ϕ	phase of oscillating parameter
Φ	functions defined by Eqs. (6a) and (10)
$\hat{\chi}$	axial displacement [m]
ψ	functions defined by Eqs. (5b) and (9)
ω	angular oscillation frequency [rad/s]

Subscripts

0	steady flow component
A	oscillating flow amplitude
b	bulk fluid property
f	fluid property
w	wall property

Other symbols

$\widehat{A}$	dimensional variable A
∇A	gradient of variable A
$\langle A\rangle$	space average of variable A
$\overline{A}$	time average of variable A
A'	phase of A relative to fixed pressure gradient
A''	phase of A relative to fixed flow rate

finite volume of a contaminant compared to molecular diffusion in a still fluid, due to the combined effect of axial convection and transverse conduction. In applications where efficient removal of the concentrate is desired, such augmented diffusion acts against the advective transport of the fluid. Chatwin [11], Watson [12] and Smith [13] determined theoretically that a similar mechanism exists in unsteady flow; however, the effect is lessened since the process cannot establish itself fully before the flow conditions are changed. In an oscillatory flow, the behaviour is governed by the frequency $Wo = (D_h/2)\sqrt{\omega/\nu}$ and dimensionless amplitude $A_0 = 2\langle\chi_A\rangle/D_h$ parameters, where χ_A is the local displacement amplitude and angled brackets denote averaging over the cross-section. The results were corroborated by experiment by Joshi et al. [14], who measured augmented diffusion in a pulsating pipe flow with $1.6 \leqslant Wo \leqslant 7.8$ and flow rate amplitude $600 \leqslant Q_A/Q_0 \leqslant 3000$. Jaeger and Kurzweg [15] determined that axial diffusion in a pulsating flow ($3.5 \leqslant Wo \leqslant 9.1$) of oxygen gas was enhanced by up to 8000 times molecular diffusion, and was proportional to the ratio of mean displacement amplitude to thermal boundary layer thickness.

The mass diffusion problem may be viewed as a generalisation of the corresponding thermal problem, which generally deals with concentration (i.e. temperature) gradients rather than finite volumes of concentration. Kurzweg [16] predicted that an equivalent thermal mechanism exists and developed a comprehensive theory for an insulated but thick-walled parallel plate geometry between hot and cold reservoirs. It was determined that enhanced axial thermal diffusivities four orders of magnitude higher than those due to molecular diffusion could be achieved. Optimal heat transfer in the axial direction was achieved when the time taken for heat to diffuse from the centre of the planar channel to the wall is equal to one half of the oscillation period. The experiments of Kurzweg and Zhao [17] validated theoretical axial heat transfer predictions in oscillatory tube flow to within a factor of a half (as explained by the change from a tube to a parallel plates geometry) over the ranges $25 \leqslant Wo \leqslant 50$ and $3 \leqslant A_0 \leqslant 20$. Effective axial conduction was enhanced by a factor of 1.79×10^4 compared with the molecular thermal diffusion value of water without accompanying net mass transport. The axial heat flux value of 292 W/cm^2 is comparable with heat pipes.

The 2-D temperature field of an oscillatory liquid flow in an insulated square channel was visualised by Ozawa and Kawamoto [18] at Wo = 4.4 and 12.2 using a thermo-sensitive liquid-crystal tracer technique. The unprocessed images were in qualitative agreement with the upward and downward peaks of isotherms, generated using a complementary two-dimensional numerical model. Further analysis of the model indicated that, over the range $2 \leqslant Pr \leqslant 100$, the thickness of the thermal boundary layer coincided with the Stokes layer thickness, suggesting that the transverse temperature gradient is formed primarily as a result of the velocity profile. While the study by Kurzweg [16] is referenced in the paper, the results are not compared with their 1-D analytical model, perhaps since the geometry of the channel was square. Also, being primarily concerned with axial heat transfer, the authors used a lumped-parameter model in conjunction with axial temperature measurements to estimate the effective thermal diffusivity over a range of frequencies $7.1 \leqslant Wo \leqslant 23.3$ and amplitudes $9 \leqslant A_0 \leqslant 36.1$.

The analytical study of Faghri et al. [19] laid the groundwork to improving our understanding of the principal mechanisms in pulsating flow. Similar to the oscillating case, the velocity and temperature oscillations interact to give a non-zero mean heat flux in the axial direction. Heat transfer was found to increase by up to 6% above the steady state value with increasing velocity amplitude