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# Image-based numerical prediction for effective thermal conductivity of heterogeneous materials: A quadtree based scaled boundary finite element method

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ABSTRACT

Integrating advantages of the quadtree technique, the SBFEM, the image-based modelling approach, and inverse analysis, a new numerical technique is presented for the evaluation of Effective Thermal Conductivity (ETC) of heterogeneous materials. The quadtree technique provides a convenient way for mesh generation, and facilitates to image-based analysis. The inconvenience of hanging nodes caused in the quadtree mesh generation can be naturally avoided by SBFEM, consequently the temperature solutions of heterogeneous materials can be determined by the combination of quadtree technique and SBFEM, and are partially regarded as 'experiments values' for equivalent homogeneous materials. Utilizing a group of such 'experiments', the ETC can be obtained by solving a group of inverse heat transfer problems of parameters identification. Numerical examples are provided to demonstrate the effectiveness of the proposed approach, and the impacts of distributions and shapes of inclusions, and volume fractions are taken into account.



## 1. Introduction

Effective thermal conductivity (ETC) is an important parameter for heterogeneous materials [1–4], the evaluation of ETC has been a topic of considerable theoretical and practical interests.

There are basically two categories of methods evaluating ETC. The first one is the analytical method, Progelhof et al. [5] provided a detailed review for analytical models, and Carson and co-workers [6,7] developed a unifying equation for five fundamental effective thermal conductivity structural models. The analytical method has good physical basis, however, there does not appear to be any single model equation that is applicable to all types of structures [6]. Another category is the numerical simulations by the computational homogenization approaches mainly based on the high fidelity finite element analysis [8–10]. Due to the complexity of heterogeneous materials, the mesh generation is challenged in numerical simulation, so the structured mesh is suggested [11,12] to decrease the computational cost of the free mesh technique.

The quadtree decomposition [13] is a hierarchical-type data structure, in which each parent is recursively divided into four children. As the element solutions for cells of the same pattern but different sizes are proportional, so this approach is high efficient, because the limited cell patterns generated can be precomputed and quickly extracted when required [14]. The quadtree decomposition is proved to be an effective mean for mesh generation in providing a simple, fast, and efficient way for data storage and retrieval [15]. In addition, the quadtree decomposition provides an effective approach for image-based analysis, which is attractive to perform virtual numerical experiments to evaluate the macro properties of heterogeneous materials [16]. As the quadtree mesh allows a fine representation of the geometry near the interface and a coarser one in the less critical area, it avoids the difficulty in computational cost for the voxel-based approach in which the mesh is built from the conversion of each voxel into a finite element [16]. However, due to the level-mismatches between adjacent elements in quadtree mesh, the 'hanging nodes' will produce, and result in a nonconformity across the interfaces and become an obstacle to impede the further application of quadtree mesh in FE analysis, although some remedial techniques have been developed [15,17].

Recently, Saputra et al. [14] presented a quadtree Scaled Boundary Finite Element Method (SBFEM) in stress analysis, where the 'hanging nodes' was effectively handled by utilizing the flexibility of polygonal elements in SBFEM [18,19]. It is of interest to integrate advantages of quadtree technique and SBFEM for the direct image-based numerical simulation for the heterogeneous heat problems.

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Because quadtree SBFEM has been demonstrated to be robust, automatic and simple, the results acquired via the simulation with various kinds of distributions and shapes of inclusions can be used as values of 'artificial measurements' for equivalent homogeneous materials, and utilized to predict ETC.

In this paper, a new numerical technique by combining the quadtree SBFEM and inverse analysis is presented to evaluate the ETC of heterogeneous materials. The technique mainly consists of two parts

(1) Reading images of heterogeneous material, generating quadtree meshes, and acquiring SBFEM based solutions with various kinds of distributions and shapes of inclusions.
(2) The results acquired via the quadtree SBFEM simulation are partially used as of 'artificial measurement values' for equivalent homogeneous materials. Utilizing a group of such 'artificial experiments' with different distributions and shapes of inclusions, the ETC can be numerically estimated by solving a group of inverse heat transfer problems of parameters identification.

The paper is structured as follows. Section 2 outlines the process of quadtree mesh generation from an image; Section 3 gives a description of building a SBFE element for steady-state heat conduction problems; Section 4 presents an approach to estimate ETC by solving a group of inverse heat conduction problems of parameter identification; and Section 5 verifies the proposed approach via numerical examples. Finally, conclusions are summarized in Section 6.

## 2. The quadtree mesh generation

According to the previous work of Saputra et al. [14], this section outlines the key process of quadtree mesh generation based on the color intensity of the pixels (2D) of an image.

The key process of quadtree decomposition mainly includes

(1) Each pixel of an image is to be represented as a square domain, as shown in Fig. 1(a), and all the colors of the pixels are stored in an image color matrix.
(2) If the difference between the maximum and minimum color intensity in a cell is larger than the color threshold, the cell is recursively divided further into four equal-sized cells, until all the cells satisfy the criterion of homogeneity or reaches the minimum edge length.
(3) During the quadtree decomposition, a balanced decomposition (2:1 rule) is used for producing only 16 possible cell patterns. Because coefficient matrix and other matrices of the cells with the same pattern but of different sizes are either identical or proportional [14], it is sufficient to treat only one cell of the same pattern and replicate the results to other cells when needed.

As shown in Fig. 1(b), the 'hanging nodes' are likely produced in the match of adjacent elements in the quadtree mesh generation. An easy treatment of these hanging nodes by SBFEM is regarding them as nodes of a new polygon element, instead of regular ones.

## 3. SBFEM based finite polygon element for heat transfer problems

Each cell generated in the quadtree decomposition is regarded as a SBFE element, as shown in Fig. 2. For heat transfer problems, the construction of coefficient matrix of a SBFE element is briefly described via several key equations, instead of detailed derivations which have been given by Wolf and Song [20].

Consider a two-dimensional heat transfer problem defined on a RUC (Representative Unit Cell) at steady-state in absence with heat generation. The governing equation is given by

$$\nabla^2 T = 0 \quad in \quad RUC \tag{1}$$

with boundary conditions

$$T = \bar{T} \quad on\ \Gamma_1 \tag{2}$$

$$-k\left(\frac{\partial T}{\partial n}\right) = q_n \quad on\ \Gamma_2 \tag{3}$$

where $T$ represents the temperature, $k$ is the thermal conductivity; $\Gamma_1 + \Gamma_2 = \Gamma$ stands for the boundary of $RUC$, $\bar{T}$ and $q_n$ are the prescribed temperature and heat flux, respectively.

In a SBFE element, the solution of temperature is described under a scaled boundary coordinate system [20], as shown in Fig. 2 where the normalized radial coordinate $\xi$ runs from the scaling center toward the boundary, and the circumferential coordinate $s$ specifies a distance around the boundary from an origin on the boundary.

The relationship between x-y and scaled boundary coordinate system is defined by

$$x = x_0 + \xi x_s(s) \tag{4}$$

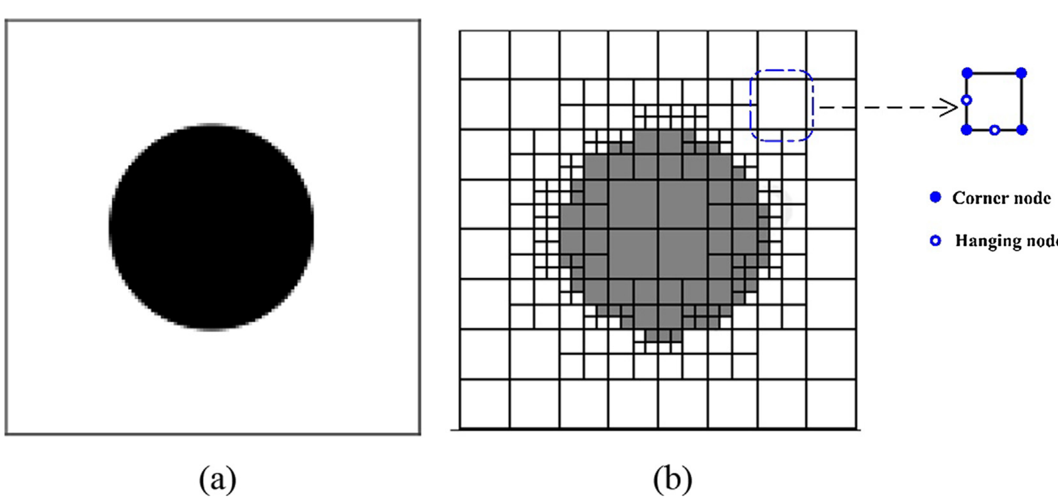


**Fig. 1.** The quadtree mesh generated from an image with an inclusion (a) Image (b) Quadtree.