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Water functionalized CuO nanoparticles filled in a partially heated trapezoidal cavity with inner heated obstacle: FEM approach

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ABSTRACT

This frame work is established to investigate the thermal management of free convection enclosed in trapezoidal cavity filled with the water based copper oxide (CuO) nanofluid. As nanoparticles volume fraction play a significant role to handle the thermal conductivity of any working fluid, so we have addressed the complex nature real world model that widely used at the industrial level and many other mechanisms. An identical trapezoidal shape cavity is placed inside the big trapezoidal cavity that have three various constraints at the surface (cold, insulated and heated). Since bottom wall of the outer cavity is partially heated so various heated portion tests are applied to analyze the influence of heat transfer within the entire cavity. Aspect ratio that depends upon the size of the inner cavity is also determine. Complete and compatible mathematical model is constructed in the form of nonlinear coupled partial differential equation. These set of equations are characterized under the law of conservation of mass, momentum and energy equation along with the restricted domain of the cavity. Koo and Kleinstreuer-Li (KKL) model is used for effective thermal conductivity and viscosity of the nanofluid. A Galerkin based Finite Element method (FEM) is implemented to attain the suitable results in term of stream function and isotherms within the restricted domain of the cavity. Results are also obtained for velocity and temperature of the nanofluid at vertically mean position of the cavity. The simulations are performed for nanoparticles volume fraction $0 \leqslant \phi \leqslant 0.2$ heated portion length $0 \leqslant L_T \leqslant 1$ aspect ratio $0.5 \leqslant AR \leqslant 3.0$, Rayleigh number $10^4 \leqslant Ra \leqslant 10^{5.7}$, and three heated conditions (cold, adiabatic and hot) for inner trapezium. It is found that flow and thermal field are getting stronger due to increase in Rayleigh number. However, fluid velocity is decreasing with increasing nanoparticles volume fraction ϕ as the fluid is getting dens. Heat transfer rate is decreasing with the increase in ϕ and L_T due to dominant convection.

1. Introduction

Natural convection heat transfer is an essential necessity in industries and engineering processes such as heating/cooling processes, solar power and chemical reactors. Particularly in the engineering applications, convection is commonly visualized in the formation of microstructures during the cooling of molten metals, and fluid flows around shrouded heat-dissipation fins, and solar ponds. It has been the subject of extensive research for the last two decades. Fetecau et al. [1] examined natural convection flow of fractional nanofluids over an isothermal vertical plate with thermal radiation. Al-Mdallal et al. [2] used natural convection for flow due to condensation on a porous vertical plate using extended homotopy perturbation method. Peric [3] studied natural convection in trapezoidal cavities. The numerical study for airflow and heat transfer for low-turbulence buoyancy-driven flow in a rectangular cavity was studied by Lyi and Hasan [4]. Prasad and Kulacki [5] studied heat transfer in rectangular porous cavity and effect of aspect ratio on flow structure numerically. They concluded that heat transfer rate increases when the aspect ratio increases. Wu and Wang [6] studied natural convection in an inclined porous cavity under time-periodic boundary conditions. Simulations were carried out recently by Aparna and Seetharamu [7] for heat transfer flow in trapezoidal cavity using finite element computational technique.

Heat transfer can be enhanced according to the industrial need by varying the boundary conditions, boundary layer turbulence and enhancement in the thermophysical properties of the working fluid. Enhancing the thermal conductivity of carrier fluid by addition of nanoparticles is found to be the best promising method in

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this regard. Initially, Maxwell [8] introduced the idea of possibility of enhancement of thermal conductivity by using small particles which has some limitations as clogging. Later on, Choi [9] concluded that nanoparticles can increase thermal conductivity of nay working fluid significantly. Kelbinski et al. [10] investigated the four techniques contribute to enhance the thermal conductivity of nanofluids: (i) nanoparticles clustering (ii) heat transport in nanoparticles (iii) Brownian motion (iv) molecular-level layering of fluid/particle interface. Aman et al. [11] studied heat transfer convection flow of Maxwell nanofluid with CNTs nanoparticles. They observed that thermal conductivity and Nusselt number of nanofluids get enhance with increasing volume fraction (Tables 2 and 3 of [11]). Aman et al. [12] studied heat transfer MHD flow of Casson nanofluids along a vertical channel and concluded that fluid flow decreases with increase in volume fraction.

The above studies were carried out on nanofluids experimentally or theoretically in different geometries. However, studies on heat transfer of nanofluids in cavities has been a subject of interest to researchers. Boulahia et al. [13] studied mixed convection heat transfer of *Cu*-water nanofluid in a square cavity with heated cylinders. Thermal management of SWCNT-water nanofluid in a partially heated trapezoidal cavity is recently investigated by Haq et al. [14]. They observed that nanofluids have higher rate of heat transfer compared to base fluid. Velocity of the fluid increases by reducing the length of heated portion but thermal field reduces. Mixed convection nanofluid in a 3D lid-driven trapezoidal cavity with flexible side surfaces and inner cylinder was analyzed by Selimefendigil et al. [15]. Alinia et al. [16] numerically examined mixed convection two phase flow of nanofluid in inclined two-sided lid-driven cavity. Melting heat transfer influence on nanofluid flow inside a cavity is analyzed by Sheikholeslami and Rokni [17]. Sheikholeslami and Sadoughi [18] studied Mesoscopic method for MHD nanofluid with different shapes of nanoparticles inside a porous cavity. Talebi et al. [19] studied mixed convection flow of nanofluid in a square lid-driven cavity. Boulahia et al. [20] numerically investigated heat transfer of nanofluid in a lid-driven cavity. Ben-Cheikh et al. [21] investigated natural convection of water-based nanofluid in a square enclosure with non-uniform heating of the bottom wall.

Natural convection in a trapezoidal enclosure filled with carbon nanotube-EG-water nanofluid was studied by Esfe et al. [22]. Job and Gunakala [23] investigated MHD mixed convection flow of nanofluids through a grooved channel with internal solid cylinders. They used Au-water and SWCNT-water nanofluid and found that groove area and shape affect fluid flow and temperature. The rate of heat transfer is proportional to Grashof number at higher Hartmann number. In case of SWCNT-water nanofluids the heat transfer rate is higher at higher Reynolds number. Kareem et al. [24] examined unsteady mixed convection heat transfer in a 3D closed lid-driven cavity. Haq et al. [25] recently investigated nanofluid in a partially heated rhombus with square cylinder. They considered *CuO*-water nanofluids inside a rhombus cavity containing a square obstacle and observed that the fluid flow, thermal field and heat transfer rate are strong for bigger values of Rayleigh number. Ismael et al. [26] studied mixed convection in square cavity filled with *CuO*-water nanofluid. Some of the recent research related to heat transfer and fluid flow is discussed in the published work [27–38].

The above literature motivated us to examine natural convection flow of *CuO*-water nanofluids in a cavity with inner heated cavity. The aim of this study is to examine numerically nanofluids flow and heat transfer in a cavity with inner heated cavity. The top wall is adiabatic and inclined walls are cold. The bottom wall is partially heated in the outer cavity. While the inner trapezium walls are considered to be heated.

2. Mathematical framework

Consider a steady two-dimensional flow of water based nanofluid with *CuO* nanoparticles inside a trapezoidal cavity with an inner heated trapezium. The enclosure is filled with Copper oxide-water nanofluid. The bottom wall of outer cavity is partially heated with constant temperature T_h while the inclined walls are cold with constant temperature T_C. The upper wall is insulated while the inner cavity is with heated walls maintained at constant temperature T_h. Natural convection is considered in this problem and flow is induced due to buoyancy force together with the external pressure gradient. Fig. 1 shows full interpretation of the geometry. In Fig. 2 is the mesh generation that helps to attain the more accurate results at various corners of the cavity. For more accuracy and better result, we increase the number of mesh at the corners of the trapezium and near the partially heated portion. The problem is modeled in the form of dimensional governing equations as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-g\rho_{nf}\beta_{nf}(T^*-T_C), \tag{3}$$

$$\left(u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial y}\right)=\alpha_{nf}\left(\frac{\partial^2 T^*}{\partial x^2}+\frac{\partial^2 T^*}{\partial y^2}\right), \tag{4}$$

where u and v are velocities along x and y directions, respectively. Here T^* is the temperature, P is the pressure and $\rho_{nf}, \mu_{nf}, \beta_{nf}, \alpha_{nf}$ are density, viscosity, thermal expansion coefficient and thermal diffusivity of nanofluid, respectively. The boundary conditions are:

Along the outer cavity:

At the left and right inclined walls:

$$T^*=T_C. \tag{5a}$$

At the bottom wall:

$$\begin{cases}\frac{\partial T^*}{\partial y}=0, & x<aL,\\ T^*=T_h, & x=(L_T)L,\\ \frac{\partial T^*}{\partial y}=0, & x>bL.\end{cases} \tag{5b}$$

At the top wall:

$$\frac{\partial T^*}{\partial y}=0. \tag{5c}$$

At all solid boundaries:

$$u=v=0. \tag{5d}$$

Along the inner cavity:

At all surfaces of the cavity:

$$T^*=T_h. \tag{5e}$$

Thermophysical properties for nanofluids are expressed in the form:

$$\begin{cases}\beta_{nf}=(1-\phi)\beta_f+\phi\beta_p,\\ \rho_{nf}=(1-\phi)\rho_f+\phi\rho_p,\\ (\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_p,\\ \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},\end{cases} \tag{6}$$