International Journal of Heat and Mass Transfer 128 (2019) 516–525

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Analytical solutions and numerical simulations of radiative property in the two-layer concentrically spherical large particle

Keyong Zhu, Shaoling Li, Yong Huang *

Beihang University, School of Aeronautical Science and Engineering, No. 37, Xueyuan Road, Beijing 100191, China

ARTICLE INFO

Article history:
Received 2 August 2018
Received in revised form 20 August 2018
Accepted 2 September 2018

Keywords:
Light scattering and absorption by particles
Absorptance
Scattering albedo
Scattering phase function
Two-layer concentrically spherical large particle

ABSTRACT

This paper quantified systematically light transfer through the two-layer concentrically spherical large particle with specular or diffuse interfaces. Analytical solutions predicted the absorptance, scattering albedo and Monte Carlo ray-tracing method was used to predict the absorptance and scattering phase function. The effects of (i) the size parameter, (ii) refractive indices, (iii) absorption indices, and (iv) the diameter ratio of the outer and inner spheres on the absorptance and scattering phase function of the two-layer concentrically spherical particle were investigated. The size parameter and absorption indices increased the absorptance for either specular or diffuse interfaces due to the stronger absorption in the two-layer concentrically spherical particle. Moreover, the refractive index of the outer sphere increased the forward scattering for specular interfaces, and scattering phase function was almost independent of the refractive indices and diameter ratio of outer and inner spheres for diffuse interfaces.

1. Introduction

Light scattering and absorption by particles are fundamental to various applications in remote sensing [1,2], atmospherical radiation [3,4], combustion [5,6] and other applications. Various numerical methods have been developed to solve light scattering by particles with nonspherical or inhomogeneous features, such as the finite difference time domain method (FDTD) [7,8], the discrete dipole approximation (DDA) [9,10] and the T-matrix method [11,12]. However, both FDTD and DDA methods are computationally time-consuming, especially for randomly oriented large particles [13,14]. The T-matrix method gives stable and convergent results up to a size parameter of 100 [15]. In this case, the ray tracing method which resorts to geometric optics approximation becomes a reasonable approach and it has been widely applied to light scattering and absorption by large particles. Yang et al. [16,17] and Xie et al. [18] developed an improved geometric optics method to simulate light scattering by large inhomogeneous ice crystals. Mikrenska and Koulev [19] adapted the Monte Carlo ray-tracing method for numerical simulation of light scattering by large particles with inclusions. Huang et al. [20,21] proposed a particle superposition model and used the Monte Carlo ray-tracing method to study radiative properties of large nonspherical particles and cenosphere.

Inhomogeneity could have significant effects on light scattering and absorption by large particles. For example, internal contamination of cloud droplets by soot may cause substantial changes in the radiative properties of liquid-water clouds [22,23], and the absorption of black carbon can be significantly enhanced due to mixing with other components [24,25]. The optical properties of core-mantle particles can change essentially when the particles uptakes water [26] or are coated by other transparent material [27]. The simplest morphological model of the inhomogeneous particles is the two-layer concentrical sphere, or the core–shell particle [28]. However, the analytical solutions of the radiative properties, such as the absorptance, for the two-layer concentrically spherical large particle based on the geometric optics approximation have not been obtained in these references.

The present paper aims to investigate systematically light transfer through the two-layer concentrically spherical large particle with specular or diffuse interfaces and give analytical expressions of the radiative properties. The effects of the size parameter, refractive indices, absorption indices and diameter ratio of the outer and inner spheres were investigated. The results will provide guidelines for the design and selection of the material and surface coating to change the radiative properties of the two-layer concentrically spherical particle in remote sensing, atmospheric radiation and other applications.

* Corresponding author.
E-mail address: huangy@buaa.edu.cn (Y. Huang).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.09.004
0017-9310/

Nomenclature

A	absorptance
D	diameter of outer sphere, μm
d	diameter of inner sphere, μm
E_n	exponential integral of order n
k	absorption index
l	path length of rays in the spheres, μm
M, m	ray number
n	refractive index
R	reflectance
T	transmittance
Greek symbols	
α	absorptance
θ	incident angle, °
φ	angle of refraction, °
λ	wavelength, μm
ρ	reflectivity
κ	absorption coefficient, μm^{-1}
τ	transmissivity in the absorbing medium
Φ	scattering phase function
χ	size parameter
ω	scattering albedo
Subscripts	
s	refers to scattering
Superscript	
d	refers to diffuse interfaces

2. Analysis

2.1. Problem statement

Fig. 1 shows the two-layer concentrically spherical large particle with outer diameter D and inner diameter d. The outer and inner spheres were featured the refractive and absorption indices denoted by n_1 and k_1, and by n_2 and k_2, respectively. The refractive and absorption indices of the surrounding air were taken as $n_0 = 1$ and $k_0 = 0$. The two-layer concentrically spherical large particle was irradiated by collimated monochromatic radiation of wavelength λ incident on its surface. Part of the incident radiation was reflected on the outer surface; the other was refracted and entered into the two-layer concentrically spherical particle. The entered radiation may experience two processes. It was reflected, transmitted, or absorbed (i) by the outer sphere, or (ii) by the inner sphere. The radiation which got out of inner sphere would repeat the two processes or leave the two-layer concentrically spherical particle by refraction.

2.2. Assumptions

To make the problem easy to handle, several assumptions were made in our analytical solution and numerical simulation:

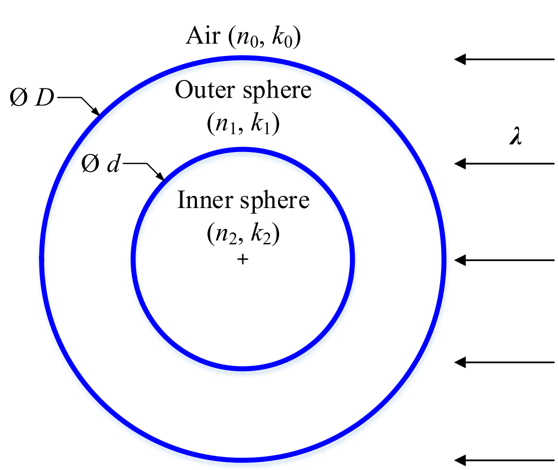

Fig. 1. Two-layer concentrically spherical large particle with outer diameter D and inner diameter d were irradiated by collimated monochromatic radiation of wavelength λ.

1. Both inner and outer spheres were homogeneous media.
2. The two interfaces (inner and outer) were supposed to be specular or diffuse simultaneously.
3. Geometrical optics prevailed and interferences, diffractions and other wave effects could be ignored, based on the much larger size of the two-layer concentrically spherical large particle than the radiation wavelength.

For specular interfaces, the reflectivity and the direction of refraction can be given by the Snell's law and Fresnel's equations [29,30],

$$n_1 \sin\theta = n_2 \sin\varphi. \tag{1}$$

Here, n_1 and n_2 are the refractive indices of the two media, respectively, and θ and φ are angles of incidence and refraction. The specular reflectivity can be written as [29,30],

$$\rho_{12} = \frac{1}{2}\left[\left(\frac{n_1\cos\theta - n_2\cos\varphi}{n_1\cos\theta + n_2\cos\varphi}\right)^2 + \left(\frac{n_1\cos\varphi - n_2\cos\theta}{n_1\cos\varphi + n_2\cos\theta}\right)^2\right]. \tag{2}$$

If $n_1 \sin\theta > n_2$, total internal reflection occurred at the interface so that $\rho_{12} = 1$. Note that the first and second subscripts of the reflectivity represent the media on the incident side and refractive side, respectively.

For diffuse interfaces, we assume that for a rough interface small enough, the optically smooth assumption still holds true, and Fresnel's equation still can be applied to calculate the small pieces of the rough interface [29,30]. Then, the diffuse reflectivity can be expressed as follows:

$$\rho_{12}^{d} = \int_0^{\pi/2} 2\rho_{12}\sin\theta\cos\theta \mathrm{d}\theta. \tag{3}$$

Finally, the transmissivity τ for an absorbing medium related to the absorption index k and the path length of the ray l is written as [29,30],

$$\tau = \exp(-4\pi k l/\lambda). \tag{4}$$

2.3. Analytical solutions

In this part, we considered two models of outer and inner interfaces in the two-layer concentrically spherical large particle, (i) two specular interfaces and (ii) two diffuse interfaces. The ray-tracing method was used to predict the absorptance and scattering albedo of the two-layer concentrically spherical large particle.