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Efficient uncertainty quantification of stochastic heat transfer problems by combination of proper orthogonal decomposition and sparse polynomial chaos expansion

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ABSTRACT

To increase the contribution and reliability of computational fluid dynamics efforts in design process of industrial equipments, it is necessary to quantify the effects of uncertainties on the system performance. Due to exponentially increment of the computational cost with number of uncertain variables for uncertainty quantification using classical polynomial chaos expansion methodology, reducing the required number of samples for uncertainty quantification is a real engineering challenge. In this paper, the proper orthogonal decomposition method based on the multifidelity approach is combined with the full and sparse polynomial chaos expansions for efficient uncertainty quantification of complex heat transfer problems with large number of random variables. The conjugate conduction heat transfer in NASA C3X cooled gas turbine blade with geometrical uncertainties and the convective heat transfer in ribbed passage with the stochastic wall heat flux boundary condition are considered as the test cases. Results of uncertainty quantification analysis in both test cases showed that proposed multi-fidelity approaches are able to produce the statistical quantities with much lower computational cost compare to the classical regression-based polynomial chaos method. It is shown that the combination of the proper orthogonal decomposition with the sparse polynomial chaos gives a computational gain at least 2 times greater than combination of the proper orthogonal decomposition with the full polynomial chaos expansion.

1. Introduction

In large number of industrial problems, uncertainties in physical properties, model parameters, operating conditions, in service degradation and manufacturing tolerances can affect the system output. An example is the cooling system of modern gas turbine blades, shown in Fig. 1, where uncertainties in mass flow, coolant properties, boundary conditions and geometry can have a large impact on the blade temperature and thus on its life. According to reference [1] the variation of the film cooling diameter due to manufacturing tolerances can reduce the engine life by 30%. This implies that to achieve a more robust design, it is necessary to include all sources of uncertainty in the design process and computational fluid dynamics (CFD) analysis. However, a single computation of cooling system of gas turbine blades based on an advanced numerical method can be extremely expensive. Consequently, the cost of quantifying a large number of random operational, geometrical etc. in such industrial applications is unacceptably high. Development of efficient uncertainty quantification (UQ) methods significantly shortens the required computational resources for the stochastic analysis of cooling system of modern gas turbine blades.

In recent years, various UQ methods such as Monte Carlo (MC) [2,3], perturbation analysis [4] and polynomial chaos expansion (PCE)[5–8] have been proposed. MC method is a sampling-based method which does not need any modification in the CFD code. In MC method, the probability space is sampled and the governing equations are solved for each sample. The numerical solutions of MC samples are then combined to make an estimate of the expectation or other statistical moments of the random solution. Despite its simplicity and independency of stochastic space size, the MC method suffers from slow convergence rate which is not desirable in industrial applications. On the other hand, the perturbation method is limited to low range of variability for input uncertain parameters which would not met in many applications. The PCE is one of the most efficient UQ analysis method which is based on spectral representation of stochastic system output [9,10]. It has been shown that the PCE is a low computational cost method in comparison to the MC [8,11] and does not have the limitations of perturbation analysis. The PCE was originally formulated for

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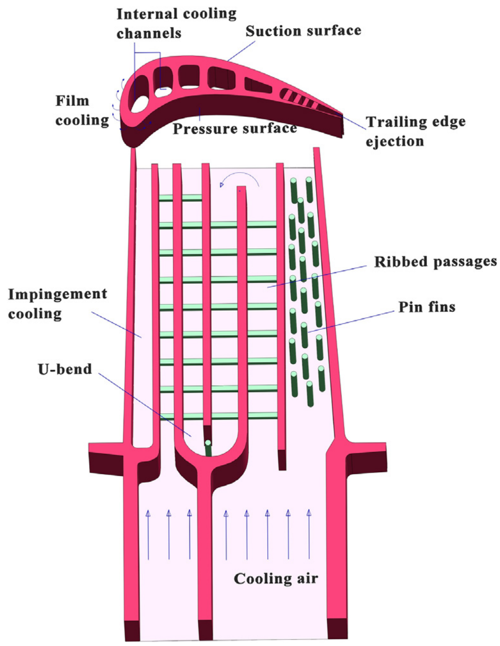

Fig. 1. Schematic of gas turbine cooling system.

stochastic problems with normally distributed random variables and Hermite polynomials as the finite-dimensional Wiener polynomial chaos [9]. Xiu and Karniadakis [7] developed the generalized PCE method and showed that for each distribution of uncertain parameters, there is an optimum orthogonal basis function for rapid convergence (e.g. Hermite polynomials for Gaussian distribution and Legendre polynomials for uniform distribution).

In recent years, the generalized PCE was implemented for UQ analysis of fluid mechanics and heat transfer problems including the cooling system of gas turbine blades. Examples of such works are D'Ammaro and Montomoli [12], Carnevale et al. [13] and Montomoli et al. [14] who performed UQ using full PCE analysis for different features of gas turbine cooling system including film cooling, pin-fin channels and high-pressure nozzle, respectively. In all these investigations, the number of uncertain parameters was limited to less than 4. However, as demonstrated earlier in Fig. 1, the number of random variables in the real gas turbine blade cooling system could be very large. This leads to the so-called "curse of dimensionality" when the classical (full) PCE is adopted. In order to reduce the computational burden, more advanced PCE strategies have to be used.

In the literature various methods are proposed to tackle the curse of dimensionality of full PCE such as sparse grid [15,16], the sparse PCE [17–19] and reduced basis method [20–22]. In the field of gas turbine heat transfer, Ahlfeld and Montomoli [23] and Mohammadi and Raisee [24] applied two efficient UQ methods, namely sparse grid and sparse PCE to deal with the curse of dimensionality, though number of uncertain parameters in these studies was also less than 10.

Model reduction is a popular technique in reducing the mathematical complexity of a model. A reduced order model could be found using a POD of the solution field. In this method, primarily the dominant modes (i.e. larger eigenvalues and corresponding eigenvectors) are found. Due to fast decay of the eigenvalues resulting from the POD, only the first few eigenvalues and eigenvectors contain most of the information, allowing an optimal basis set of small size.

As described in [25], the reduced order models can be divided into two categories, namely intrusive and non-intrusive methods, based on their dependency on the mathematical description (i.e. governing equations) of the problem. Due to intrusiveness, the former retains much of the characteristics from the original system. However, this method requires modification of the deterministic codes which is difficult, expensive and time-consuming for many engineering problems. Furthermore, the source codes of most commercial softwares are not accessible and thus it is not feasible to implement the intrusive approach in such deterministic softwares. For these reasons, here we focused on non-intrusive POD methodology for UQ analysis. In non-intrusive approach, the deterministic code is considered as a "black-box" and reduced order approximation is obtained based on deterministic evaluations.

Xiao et al. [26] proposed the non-intrusive reduced order models in combination with radial basis function for the Navier-Stokes equations. They applied the model in two CFD problems including lock exchange problem and flow past a cylinder. The obtained results showed that the proposed method reduced the computational cost by at least two orders of magnitude while the level of accuracy was similar to the corresponding high-fidelity models. They also, applied the model for fluid-structure interactions (FSI) including flow past a cylinder, free-falling of a square cylinder in water and vortex-induced vibration of an elastic beam. Results showed the great advantages of the proposed method in FSI problems. More specifically, the CPU time required for the non-intrusive reduced order model with the maximum number of POD basis was reported to be 0.1–7% of that required for the full simulation [27]. In the literature, combination of the POD with other efficient methods such as Kriging or analysis of variance (ANOVA) has been implemented for UQ analysis. Margheri and Sagaut [28] developed an efficient method by combination of anchored-ANOVA and the POD/Kriging methods. They successfully implemented this method in four high-fidelity CFD problems.

The POD method requires the covariance of the stochastic solution field. It is known that the two sources of error in the stochastic problems, namely the spatial and random discretization errors are more or less decoupled [29]. This implies that the covariance function may be found using the low-fidelity approximation of the desired field. Doostan et al. [29], using this concept proposed an intrusive reduced basis model to deal with the curse of dimensionality. They applied it successfully to a stochastic 2D solid mechanics problem. The main idea of this approach is to compute the optimal orthogonal basis using inexpensive calculations on a coarse mesh and then use these basis for the computation on a fine mesh. The proposed intrusive method in [29] cannot be coupled easily with the commercial flow solvers to tackle complex CFD problems. In Raisee et al. [21], this idea is extended to non-intrusive PCE and subsequently applied to a complex mathematical function and two simple CFD problems. The same research group in [30], applied the developed non-intrusive POD-based method to 2D and 3D turbulent Navier-Stokes including: (1) flow over a 2D RAE2822 airfoil and (2) the NASA Rotor 37, a transonic axial flow compressor. They showed that the proposed model is able to successfully reproduce the statistical results with good accuracy but with 5–10 times lower computational cost in comparison to the classical PC.

The above mentioned intrusive and non-intrusive models in [29,21] can be considered as a multi-fidelity model, where combination of large number low-fidelity with a few high-fidelity evaluations gives results with the same accuracy in comparison to expensive high-fidelity evaluations. This approach was firstly introduced by Ng and Eldred [31] for computing the coefficients of PCE using sparse grids. Salehi et al. [32] combined the multi-fidelity approach with the sparse PCE to make an efficient method which has the capability to reproduce accurate results with much lower computational cost than the classical full PCE and sparse PCE methods.