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A new algorithm for solving an inverse transient heat conduction problem by dividing a complex domain into parts

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A B S T R A C T

The purpose of this work is to create a new algorithm for solving an inverse transient heat conduction problem in any extended complex domain. If the problem cannot be solved in the entire region under analysis, the domain is divided into parts and free boundary conditions are introduced between them. Then the solution is carried out in subsequent parts of the field. Depending on the shape and dimensions of the analysed part, an appropriate location of the temperature sensors, the amount and size of the control volumes, the time step and a convenient way to filter the temperature distribution over time and space can be proposed. Such actions make it possible to achieve better stabilization of the inverse method and obtain the solution in the whole area. The proposed procedure in subsequent parts of the domain belongs to the group of space marching methods. The analysis starts on the surface where temperature sensors are located and marches through space sequentially to the surface with an unknown boundary condition.

The proposed algorithm can be used to optimize the power unit start-up and shutdown operation. It may also enable a reduction in the heat loss arising during the process and extend the power unit life. The presented procedure can be applied in monitoring systems working both in conventional and nuclear power plants.

1. Introduction

The thermomechanical analysis plays an important role in the design and operation of equipment operating at high temperature differences. Such analyses are carried out in microelectronics [1], construction [2], power industry [3] and in research on nuclear fusion [4]. The fundamental problem in this type of investigations is the difficulty in determining some of the thermal boundary conditions. In the power unit components, the boundary condition on internal surfaces in contact with the fluid is very difficult to establish. It can be estimated through a numerical analysis of the phenomena that take place in the flowing fluid using the control-volume finite element method [5]. Another way to approximate the distribution of temperatures even if the boundary condition is unknown is to solve the inverse heat conduction problem (IHCP) in the object under analysis [6]. Using inverse methods, it is possible to calculate the steady- or the transient-state temperature distribution in an element based on measured temperature histories in selected spatial points. For components with simple and regular shapes, with the material properties assumed as temperature-independent, exact methods of solving the inverse heat conduction problem can be applied [7–9].

Solving the IHCP in bodies with complex shapes or with temperature-dependent thermal properties requires numerical methods. There are several studies devoted to one-dimensional [10,11] and two-dimensional problems [12]. A three-dimensional ill-posed boundary inverse problem can be solved by means of an iterative regularization method [13]. A general method for solving multidimensional inverse heat conduction problems is shown in [14]. A new method for identification of the refractory lining state and the heat transfer coefficient between the FCC zeolite catalyst particles and the regenerator walls can be found in [15]. A three-dimensional numerical model for inverse determination of the heat flux and the heat transfer coefficient distributions over the metal plate surface cooled by water is shown in [16]. However, the methods mentioned above are useful only if applied to elements with a simple geometry.

An on-line inverse method combined with the finite-element scheme is proposed in [17] to estimate the unknown heat flux on the nozzle throat–insert inner contour. The finite-element scheme can easily approximate the irregularly shaped boundary. An inverse method which can be used to solve inverse heat conduction problems in thick-walled complex-shaped elements can be

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found in [18]. The formulation of two simple methods of solving nonlinear inverse heat conduction problems in a complex-shaped component without the simplifying assumption of thermal insulation on the analysed domain boundaries is shown in [19].

The presented methods solve the inverse problem in a selected part of the component. The analysed area has to be small enough, otherwise the solution becomes unstable. The unknown boundary conditions make the inverse problem ill-posed and the calculated temperatures become very sensitive to errors made while calculating "measured" temperatures or performing real-time measurements. The errors can result in temperature oscillations, which can make the solution unstable. Many different techniques are proposed in literature to overcome such difficulties, including smoothing digital filters [18], regularization [20], future time steps [21], and the domain decomposition method [21]. The inverse solution can be stabilized by dividing the space domain into several sub-domains and assuming that both boundaries perpendicular to the unknown boundary are insulated [21]. Unfortunately, with the proposed division of the area into sub-domains towards the surface with an unknown boundary condition, it is still impossible to increase the area of the analysis along that surface. For this reason, the area under analysis has to be sufficiently small.

The purpose of this work is to formulate a new algorithm for solving an inverse transient heat conduction problem in any extended complex domain. If the problem cannot be solved in the entire region under analysis, the domain is divided into parts and free boundary conditions are introduced between them. Then the solution is carried out in subsequent parts of the field. Depending on the shape and dimensions of the analysed part, an appropriate location of the temperature sensors, the amount and size of the control volumes, the time step and a convenient way to filter the temperature distribution over time and space can be proposed. Such actions make it possible to achieve better stabilization of the inverse method and obtain the solution in the whole area. The proposed procedure in subsequent parts of the domain belongs to the group of space marching methods. The analysis starts on the surface where temperature sensors are located and marches through space sequentially to the surface with an unknown boundary condition.

2. Formulation of the problem

Assume simplifying the three-dimensional heat conduction analysis to a two-dimensional heat conduction problem. The equation governing the transient heat conduction problem has the following form [22]:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + q_V, \tag{1}$$

where $\mathbf{q}$ is the heat flux vector defined by the Fourier law and q_V is the heat generation rate per a unit volume,

$$\mathbf{q} = -\mathbf{D}\,\nabla T. \tag{2}$$

$\mathbf{D}$ is the conductivity matrix and in a two-dimensional case it is defined as:

$$\mathbf{D} = \begin{bmatrix} k_x(T) & 0 \\ 0 & k_y(T) \end{bmatrix} \tag{3}$$

For isotropic materials $k_x(T) = k_y(T) = k(T)$. The material properties (c – specific heat, ρ – density, k – thermal conductivity) are defined as known functions of temperature.

In order to solve the problem, appropriate initial and boundary conditions have to be established. The initial condition, or the Cauchy condition, is the body temperature at the first moment: $t_0 = 0$ s.

$$T(\mathbf{r}, t)|_{t_0=0} = T_0(\mathbf{r}) \tag{4}$$

where $\mathbf{r}$ is the position vector.

Three of the most often used boundary conditions, of the first, second and third kind, can be assigned to the body boundary

$$T|_{\Gamma_T} = T_b \tag{5}$$

$$(\mathbf{D}\,\nabla T \cdot \mathbf{n})|_{\Gamma_q} = q_b \tag{6}$$

$$(\mathbf{D}\,\nabla T \cdot \mathbf{n})|_{\Gamma_h} = h\left(T_m - T|_{\Gamma_h}\right) \tag{7}$$

where

- $\mathbf{n}$ – unit outward vector normal to boundary Γ,
- T_b – temperature set on the body boundary Γ_T,
- q_b – heat flux set on the body boundary Γ_q,
- h – heat transfer coefficient set on the body boundary Γ_h,
- T_m – temperature of the medium.

The phenomenon described by Eqs. (1)–(7) where boundary conditions are specified over the entire boundary is called a direct transient heat conduction problem (cf. Fig. 1).

If a part of the boundary has an unknown condition, as presented in Fig. 2, the problem becomes ill-posed. In such a case, additional temperature measurements are needed in the analysis to solve it.

$$f_i(t) = T(r_i) \quad i = 1, \ldots, N_T \tag{8}$$

The measurement errors are smaller if temperature is measured on the surface rather than in the middle of the wall thickness. It is proposed that the temperature measurement points should be located on the surface with a known boundary condition.

Solving inverse problems is more difficult compared to direct ones. There are frequently considerable oscillations in the body determined temperature that do not arise if the phenomenon takes place in real conditions. Sometimes, the oscillations may make the calculations completely unstable. If the problem cannot be solved in the entire region under analysis, or if the solution error is large, the domain can be divided into parts as shown in Fig. 3. Then the solution is carried out in subsequent parts of the field with free boundary conditions introduced between them. The Free (Open) Boundary Condition (FBC, OBC) was proposed by Papanastasiou et al. [23] to handle truncated domains with arbitrarily imposed boundaries where the outflow conditions are unknown. It can be seen in Fig. 3 that the distance between the surface with an unknown boundary condition and the measuring sensors is the largest for the third part and the smallest – for the first. It should be expected that the inverse procedure carried out for the entire domain will produce a much worse solution for part 3 than for the other parts. The solution accuracy can be improved by conducting the analyses separately for each subsequent part.

The control-volume finite element method is applied to solve problems that occur in elements with complex geometries [24].

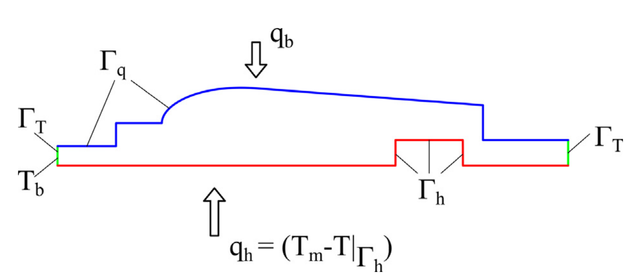

Fig. 1. A direct transient heat conduction problem.