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Modelling the radiation efficiency of orthotropic cross-laminated timber plates with simply-supported boundaries

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ABSTRACT

In this paper two prediction models to evaluate the radiation efficiency of orthotropic plates, developed with different approaches, are presented. A sound radiation model, based on an analytical/modal approach, is developed for a thin orthotropic plate, with the principal directions aligned with the edges. The model allows to consider the contribution of each mode, either resonant or non-resonant, as well as the influence of fluid loading on the plate dynamic response and on sound radiation. Moreover, a statistical model to evaluate the average radiation efficiency, based on a non-modal approach, which only considers the contribution of resonant modes, is presented. These two models have been used in order to predict the radiation efficiency of orthotropic cross-laminated timber (CLT) plates. CLT is an engineered wood material constituted by an odd number of lumber beams glued together, which have become very popular in the last twenty years in the building construction market. Due to their layered structure, CLT plates might exhibit a highly orthotropic behaviour. Both prediction models are validated by comparing the simulated results with the experimental radiation efficiency, obtained by means of vibro-acoustic measurements on three CLT plates. Finally, the influence of fluid loading on sound power radiated by CLT plates is investigated.

1. Introduction

Noise reduction is nowadays a main concern as much in the automotive or aerospace industry as in building construction and in many other fields. In order to design structures that provide good sound insulation it is fundamental to characterise how the vibrating elements radiate sound. Sound radiation has been the object of an increasing interest during the last half century and the physics behind this mechanism is well known. However, from an engineering point of view, the computation of the sound power radiated by a vibrating surface is still a highly demanding task compared to pure vibrational problems. In order to provide reliable alternatives to finite elements (FE) and boundary elements methods (BEM), which usually require a considerable computational effort, many formulations to predict the radiated sound power have been proposed by several authors. These prediction models provide approximated results with wide-ranging levels of accuracy. They have been developed by using different approaches and under different basic assumptions, upon which depends their suitability for each specific problem. Based on a *non-modal approach*, the modal-average radiation efficiency due to the contribution of the resonant vibrational field can be evaluated [1]. On the other hand, using a *modal approach* the radiation efficiency is approximated by taking into account all the in-vacuo single modes within the frequency range [2]. In order to consider the influence of cross-modal coupling, different *analytical formulations* have been developed using either the far field [3], or the near-field approach [4]; the latter also takes into account the fluid loading effect. An extensive and more detailed bibliographic analysis of the prediction models to compute sound radiation was presented by Atalla and Nicolas in 1994 [5]. Later in the same decade Nelisse [6] proposed a generalized model for the acoustic radiation from baffled and unbaffled homogeneous plates, with arbitrary boundary conditions. The same approach was also used by Foin to develop a tool to predict the acoustic and structural vibration response of sandwich plates [7]. More recently Mejdi and Atalla presented a semi-analytical model to numerically investigate the vibro-acoustic response of stiffened plates [8], while Legault analysed orthogonally ribbed plates by means of a periodic theory [9]. Rhazi and Atalla used simple and quick tools, such as statistical energy analysis and the transfer matrix method, to estimate the vibro-acoustic

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response of mechanically-excited multilayer structures [10]. Davy developed a two dimensional strip analytic approximation to compute the forced radiation efficiency of acoustically excited finite size panels [11]. Davy also presented an approximation method to calculate both the real and the imaginary part of the single-side specific forced radiation impedance of a rectangular panel [12]. The possibility to consider both the resonant and non resonant contributions, in the case of an acoustically excited plate, and the near-field contribution in the case of mechanical point excitation, was also introduced by Davy in a recently published paper [13].

Two models are presented in this paper; they have been implemented in order to estimate the radiation efficiency of mechanically excited orthotropic panels, such as cross-laminated timber plates used in buildings. Cross-laminated timber solid wood panels, commonly known as CLT, consist of an odd number of layers of lumber beams glued together, alternating the fibres orientation of adjoining plies orthogonally. This engineered wood material has gained a growing success in the construction market over the last two decades, especially in Europe and North America. In fact in recent years, CLT has also attracted the interest of acousticians and researchers who have carried out experimental investigations on these structures [14–18]. Nowadays, CLT structures represent a valuable alternative to traditional construction materials. They provide good structural stability, fulfil the safety requirements and allow to reduce construction time, since they can be completely prefabricated and then assembled at the construction site. The drawback of this construction technology is arguably the poor sound insulation provided by CLT panels, due to their low density combined with a relatively high stiffness. During the design process it is necessary to acoustically optimize the CLT elements in order to improve the sound insulation performace and meet the acoustic requirements for buildings [19]. Due to their layered sub-structure and the properties of the wood material, CLT plates generally exhibit an orthotropic behaviour [20,21], which means that they have different elastic properties along mutually perpendicular directions. CLT panels can be investigated as 3D orthotropic plates. This approach, however, would involve a rather tedious and complex analysis with nine independent elastic constants to be known: i.e. three elastic moduli and three Poisson's ratios associated with the principal directions and three shear moduli. In order to define more usable models, the thin plate theory is here adopted, since it greatly simplifies the problem, describing the orthotropic constitutive relationship by means of only five independent constants. However, in order to take into account the influence of rotatory inertia and shear deformation, which are neglected in the thin plate theory but might be significant especially in the high frequency range, the CLT panels are described by means of apparent frequency-dependent elastic properties, as further discussed below.

In the next section the numerical models to predict the radiation efficiency of orthotropic plates are introduced. At first, an analytical formulation for a thin orthotropic plate is derived, by following the general approach, based on a variational formulation, proposed by Nelisse [6]. Then a simplified modal-average approach to compute the orthotropic radiation efficiency, based on more restrictive assumptions, is described. Both models have been validated with the experimental radiation efficiency evaluated for three different CLT plates, as described in Section 3. The main results are presented and discussed in Section 4.

2. Prediction models for an orthotropic plate

The radiation efficiency σ is defined as the ratio between the sound power W_{rad} actually radiated by a vibrating elastic structure and the sound power that would theoretically be radiated by a rigid piston of equal surface area S vibrating with the same mean square velocity $\langle v^2\rangle_{s,t}$, where the subscript $_{s,t}$ indicates time and spatial average, multiplied by the characteristic air impedance $Z_0 = \rho_0 c_0$:

$$\sigma = \frac{W_{rad}}{\rho_0 c_0 S \langle v^2\rangle_{s,t}}. \tag{1}$$

where ρ_0 is the density of the fluid medium and c_0 the speed of sound within the fluid. This acoustic descriptor, characterising the capability of a vibrating structure to transfer the vibrational energy to the surrounding fluid as sound energy, represents important input data, required in the greatest part of building acoustics prediction models [22–26]. In this section two models to evaluate the radiation efficiency of an orthotropic rectangular plate are presented. They are based on different assumptions and developed following distinct approaches. An analytical/modal-based approach is derived, either considering or neglecting the influence of the fluid loading. Then a modal-average model, which may be useful within the statistical energy analysis (SEA) framework, is presented.

Both models assume the validity of thin plate theory. However, as the frequency increases and the structural wavelength approaches the panel thickness, rotational inertia and shear deformation effects start to have a significant influence on the plate dynamics. For this reason, apparent frequency-dependent stiffness properties have been introduced in order to adopt low-order theories while considering several effects which take part in the flexural motion, such as shear deformation, rotatory inertia, viscoelasticity and the layered substructure. The possibility to adopt such a homogenisation approach, commonly used to investigate sandwich structures [27–29], also to CLT panels has already been successfully investigated and discussed by other authors in previous studies [20,30].

2.1. Modal based radiation and fluid-loading

Let us consider a rectangular thin orthotropic plate, with the principal axes aligned with the edges, lying in the $x-y$ plane and inserted in a coplanar rigid baffle, as shown in Fig 1. The equation of motion of such a thin orthotropic plate, undergoing free flexural vibrations, is governed by the fourth-order in space and second-order in time differential equation:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2B\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2}. \tag{2}$$

The bending stiffness along the principal directions, D_x and D_y, and the effective torsional stiffness B, are a function of elastic and in-plane shear moduli E_x, E_y and G_{xy}:

$$D_x = \frac{E_x h^3}{12(1-\nu_{xy}\nu_{yx})}; \quad D_y = \frac{E_y h^3}{12(1-\nu_{xy}\nu_{yx})}; \tag{3}$$

$$B = \frac{\nu_{xy} D_y}{2} + \frac{\nu_{yx} D_x}{2} + 2G_{xy}\frac{h^3}{12}. \tag{4}$$

The elastic constants ν_{xy} and ν_{yx} are related to the geometrical configuration of the orthotropic plate [31]. According to Betti's reciprocal theorem, the bending stiffness along the two principal directions satisfies the relationship [32]:

$$\nu_{yx} D_x = \nu_{xy} D_y. \tag{5}$$

A sound radiation model for an orthotropic thin plate has been developed using a general approach based on Hamilton's variational principle. The solution for the plate's transverse displacement w can be derived following the generalised approach