Applied Acoustics 143 (2019) 59–65

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Generalized power law for predicting the air flow resistivity of thermocompressed fibrous materials and open cell foams

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ARTICLE INFO

Article history:
Received 1 June 2018
Received in revised form 15 August 2018
Accepted 28 August 2018

Keywords:
Power law
Sound absorbing
Thermocompression
Air flow resistivity
Fibrous material
Open cell foam

ABSTRACT

The compression of sound absorbing materials can change significantly their acoustical properties. It is commonly admitted that the air flow resistivity is the main parameter governing the acoustical efficiency. Several authors propose empirical laws to predict the resistivity as function of the compression rate. The models are based on a power function with a power related to the kind of material. In this paper, we propose a generalized power law to calculate the air flow resistivity of compressed porous materials with high porosity ($\geqslant 0.9$). The power is here related to the material initial porosity. The influence of fibres orientation distribution is also addressed. The proposed formula shows a good agreement with the measurements performed on several types of porous materials (four fibrous materials and two open cell foams).

1. Introduction

Porous materials such as fibrous or foams are widely used in almost every areas of noise control engineering such as automotive, building and aerospace. In some applications, they are compressed into thin non uniform panels. This compression modifies the bulk material density, the porosity, the air flow resistivity and consequently, the acoustical properties. Among these parameters, the resistivity is admitted to be the main parameter governing the acoustical properties [1–6].

To estimate analytically the air flow resistivity of porous materials, two approaches are encountered.

The first approach is analytical: simplified geometrical models of the porous material are used to derive an analytical expression of the resistivity. For fibrous materials, a general equation is proposed by Tamayol [9] for a periodic regular arrangement of cylinders with a parallel or perpendicular flow. Tarnow [10,11] introduced a Voronoi distribution to model a random network of parallel cylinders. For more complex microgeometries, like foams, only numerical models are suitable [7,8].

The second approach is experimental: a large number of materials are tested and empirical laws for the resistivity are then proposed (see Table 1). Kozeny–Carman [12,13] first developed a relation to calculate the resistivity of a granular media. Other authors have applied this relation for modeling fibrous materials [14]. In these works the flow resistivity is related to the porosity, the particle size and a factor K_c obtained from the measurements. Nichole [15] related the air flow resistance R to the thickness of the material L, the surface density S, and the cross-sectional radius of the fibres a by a power law. Bies and Hansen [16] presented a simple model to calculate the resistivity of fibrous materials with the fibre diameter d, the bulk density ρ_m, and two geometrical constants K_1 and K_2. They pointed out that the equation was suitable for fibres with uniform diameters. Garai and Pompoli [17] extended the equation of Bies and Hansen to polyester fibres with larger and more dispersed diameters and densities. Kino [18–20] applied the law of Bies and Hansen to different porous materials (polyester material, glass wool and melamine foam). They modified the values of the two constants K_1 and K_2 to fit with their measurements. However, no general law has been given so far to derive these constants for any kind of porous material.

Castagnède et al. [21,22] studied the relationship between the physical parameters in the equivalent fluid model and the compression rate n, n being defined as the ratio between the nominal thickness $h^{(1)}$ and the compressed thickness $h^{(n)}$. They proposed a linear and a quadratic law to predict the resistivity after compression, but their models are limited to low compression rates $n < 2$. Lei et al. [23] proposed new formulas to predict the variations of six physical parameters suitable for high compression rates (n up to 10). Their model accounts for the variation of fibres orientation

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https://doi.org/10.1016/j.apacoust.2018.08.029
0003-682X/

Table 1
Empirical model for the resistivity of porous materials.

Authors	Model	Type of materials
Kozeny, Carman 1937	$\sigma = \frac{K_c}{d^2}\frac{(1-\phi)^2}{\phi^3}$	Granular media $0.9 < \phi < 1.0$
Nichols, Jr RH 1947	$R = K\frac{S^{(1+x)}}{L^x a^2}$ $0.3 \leqslant x \leqslant 1$	Fibrous materials $0.9 < \phi < 1.0$
Bies, Hansen 1980	$\sigma = K_1 \cdot \rho_m^{K_2}/d^2$ $K_2 = 1.53$ $K_1 = 3.18 \times 10^{-9}$	Glass fibres $1 < d < 15$ μm
Garai, Pompoli 2005	$K_2 = 1.404$ $K_1 = 28.3 \times 10^{-9}$	Polyester fibres $18 < d < 48$ μm
Kino, Ueno 2007	$K_2 = 1.53$ $K_1 = 15 \times 10^{-9}$	Polyester fibres $14.2 < d < 39$ μm
Kino, Ueno 2008	$K_2 = 1.53$ $K_1 = 11.5 \times 10^{-9}$	Melamine foam $100 < d < 200$ μm
Kino, Ueno 2009	$K_2 = 1.53$ $K_1 = 8 \times 10^{-9}$	Melamine foam $150 < d < 300$ μm

due to the compression, on the basis of the works of Tarnow [10,11] and Tamayol [9].

In this paper, we propose a generalized power law to predict the air flow resistivity of a compressed porous material from its initial porosity and resistivity, without adjusting any empirical coefficient. First, the power law is given and a polynomial expression of the exponent K_2 is derived from the analytic models presented in our previous paper [23]. The polynomial relation are function of the initial porosity, and are given for different initial fibre orientations and different fibre arrangements (regular or random). Finally, the predictions are compared to the measurements of several types of porous materials (glass wool, polyester fibres, foams) and compression rates.

2. Theory

2.1. Power law for air flow resistivity

The resistivity proposed by Bies and Hansen is presented in the following form:

$$\sigma = K_1 \rho_m^{K_2}/d^2, \tag{1}$$

where ρ_m is the mass density, d is the fibre diameter, K_1 is a constant, characteristic of a particular material. The value K_2 is determined from the construction of the material which is linked to the arrangement of fibre and their orientation in the material. When the compression occurs, K_1 and d are considered not to change, while the mass density after compression is $\rho_m^{(n)} = n\rho_m^{(1)}$. Together with Eqs. (1) the air flow resistivity at compression rate n writes:

$$\sigma^{(n)} = n^{K_2}\sigma^{(1)}. \tag{2}$$

For a material having a porosity between 0.9 and 1, the value K_2 varies from 1.3 to 2 [15]. However no explicit relation is established between the porosity and the constant K_2 in the literature. This is the main objective of this paper.

2.2. Determination of the constant K_2

The exponent K_2 can be related to the initial porosity of the material by best fitting the power law Eqs. (2) with the resistivity derived analytically in Ref. [23]. The analytic resistivity of a compressed material $\sigma^{(n)}$ is recalled here for two fibres arrangements:

$$\sigma_r^{(n)} = n\sigma^{(1)}\frac{[0.64\ln(1/(1-\phi^{(1)}))-\phi^{(1)}+0.263]}{[0.64\ln(1/(1-\phi^{(n)}))-\phi^{(n)}+0.263]} \cdots \frac{\sum_{i=1}^m f_i(1+\tan^2\theta_i^{(1)})/(2+\tan^2\theta_i^{(1)})}{\sum_{i=1}^m f_i(n^2+\tan^2\theta_i^{(1)})/(2n^2+\tan^2\theta_i^{(1)})} \tag{3}$$

$$\sigma_s^{(n)} = n\sigma^{(1)}\frac{\ln(1-\phi^{(1)})-2(1-\phi^{(1)})+1.479+(1-\phi^{(1)})^2/2}{\ln(1-\phi^{(n)})-2(1-\phi^{(n)})+1.479+(1-\phi^{(n)})^2/2} \times \frac{\sum_{i=1}^m f_i(1+\tan^2\theta_i^{(1)})/(2+\tan^2\theta_i^{(1)})}{\sum_{i=1}^m f_i(n^2+\tan^2\theta_i^{(1)})/(2n^2+\tan^2\theta_i^{(1)})}, \tag{4}$$

where the subscript s, r indicates the arrangement of fibres either a square or a random array respectively, $\phi^{(1)}$ and $\phi^{(n)}$ are the porosity of the material before and after compression, $\theta_i^{(1)}$ is the fibre initial orientation angle, f_i is the initial probability density function so that $f(\theta_i^{(1)}) = f_i$, and m is number of angular partitions between 0 and $\pi/2$.

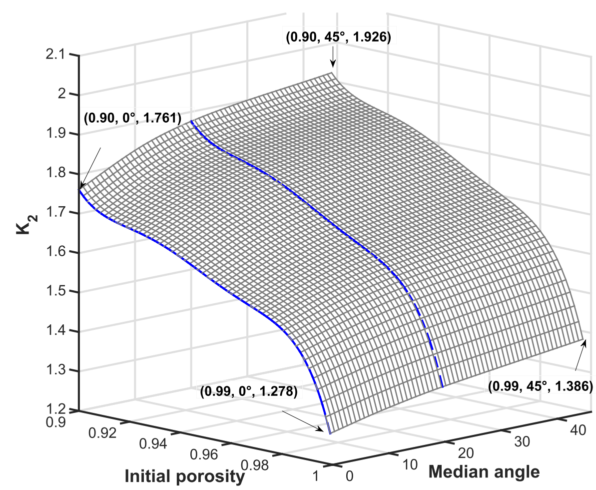

Fig. 1. Exponent K_2 drawn as a function of the initial porosity $\phi^{(1)}$ and initial angular distribution represented by the median angle $\tilde{\theta}^{(1)}$ for a random arrangement of fibre (Eq. (3)). The two solid blue lines represent the polynomial function for $\tilde{\theta}^{(1)} = 0^\circ$ (Eq. (8)) and $\tilde{\theta}^{(1)} = 20^\circ$ (Eq. (6)) plotted in figure and respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

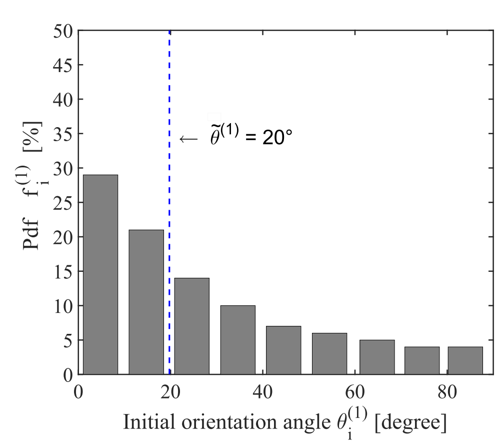

Fig. 2. Generated angular distribution for fibrous material with the median angle $\tilde{\theta}^{(1)} = 20^\circ$ (the dash line).