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Asymptotic solution for interface crack between two materials governed by dipolar gradient elasticity

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ABSTRACT

An asymptotic solution for interface crack between mismatched materials that obey a special form of linear isotropic strain gradient elasticity under conditions of plane strain is developed. It is shown that the asymptotic solution depends in a complicated manner both on a mismatch of elastic properties and a mismatch of the internal length scale parameter. Numerical analysis shows that the mismatch of elastic moduli and the gradient coefficient *c* respectively lift the degeneracy of the exponent *p* that is characteristic for a crack in homogeneous material. The total energy release rate G^{int} due to a crack extension along the interface is derived generalizing the classical virtual crack closure method. The reciprocal work contour integral method for the evaluation of amplitude factors in the asymptotic expansion is extended to interface crack in the linear isotropic strain gradient elasticity.

1. Introduction

The paper deals with the determination of the asymptotic displacement, strain and stress fields in the vicinity of the interface crack between two mismatched materials governed by dipolar gradient elasticity which by introducing intrinsic length scale allows predicting the scale effects observed experimentally. The gradient elasticity and, in general, higher order continua theories have recently attracted the attention of researchers in various disciplines due to the ability of the higher-order or nonlocal terms to model phenomena which cannot be described by classical elasticity since it does not include an internal length scale in its constitutive structure. Among these phenomena belong for example the occurrence of size effects (i.e. the dependence of strength or other macroscopic properties on specimen size), as well as fracture and/or interface processes where the detailed "non-singular" or "continuous" distribution of stress and strain fields near the crack tip or *the interface* is of prime importance. Currently an increasing effort to capture effects of microstructure in the framework of generalized continuum theories is driven by emerging of new materials and devices with the characteristic size lengths comparable to the lengths of material microstructure. The intention is to extend the range of applicability of the continuum concept and thus to contribute to bridging the gap between classical continuum theories and atomic-lattice theories. A major reason of this effort is that molecular dynamics and/or quantum mechanics based simulations require tremendous computation resources. A general higher-order elastic theory was proposed by Mindlin [1]. For practical purposes, Mindlin also formulated three simplified versions of the general isotropic theory, utilizing only two material and five internal length-scale constants in the final constitutive relation rather than 18 used in Mindlin's initial model. According to the inherent form of the strain energy density these three versions are specified as Form I, II and III. In the case of the Form I is the strain energy density a quadratic form of the classical strains and the second gradient of displacement. In the case of the Form II is the strain energy density a quadratic form of the classical strains and the gradient of strains and in the case of the Form III is the strain energy density a quadratic

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Nomenclature

Mathematical symbols

$\otimes$ tensor product
$\cdot$ scalar product
$D = \mathbf{n}\cdot\nabla$ normal derivative on the contour with the normal unit vector $\mathbf{n}$
∂_k partial derivative with respect to the coordinate k
∇ gradient operator
∇^2 Laplacian operator
$\overset{s}{\nabla} = \nabla - \mathbf{n}D$ surface gradient operator on the contour with the normal unit vector $\mathbf{n}$
$i = (-1)^{1/2}$ complex unit
$\mathbf{I}$ unit dyadic tensor
$\widehat{\mathbf{I}}$ anti-diagonal unit dyadic tensor
(r,φ), (x,y) polar and Cartesian coordinates
$\Re$, $\Im$ real and imaginary part of a complex expression
$[\![f]\!]$ jump of a function f across the interface
$\Gamma(.)$ Gamma function

Greek symbols

β Dundurs parameter
Γ closed contour encircling the crack tip
ε imaginary part of the singularity exponent in the classical elasticity
$\boldsymbol{\varepsilon}^{(J)}$, $\varepsilon_{kl}^{(J)}$ strain tensor and its kl component
κ_J Kolosov's constant
λ_J Lamé constant
μ_J shear modulus
ν_J Poisson's ratio
$\sigma_{kl}^{(J)}$ kl -stress component in the classical elasticity
$\boldsymbol{\tau}^{(J)}$, $\tau_{kl}^{(J)}$ monopolar stress tensor and its kl -component in the gradient elasticity
$\underset{\sim}{\boldsymbol{\tau}}^{(J)}$ angular functions of the asymptotic monopolar stress vector $\boldsymbol{\tau}^{(J)}$
ψ, ψ_k phase angle

Latin symbols

Δa finite crack extension
$A_{1\ldots4}^{(J)}$, $B_{1\ldots4}^{(J)}$ elements of the eigenvector $[\mathbf{a}^{(J)},\mathbf{b}^{(J)}]^T$
$\mathbf{A}^{(J)}$, $\mathbf{B}^{(J)}$ matrices of the angular functions in the expression for the asymptotic displacements $\mathbf{u}^{(J)}$
$\mathbf{A}_{,\varphi}^{(J)}$, $\mathbf{B}_{,\varphi}^{(J)}$ derivative of the elements of the matrices $\mathbf{A}^{(J)}$ and $\mathbf{B}^{(J)}$ with respect to φ
$[\mathbf{a}^{(J)},\mathbf{b}^{(J)}]^T$ eigenvector in the eigenvalue problem (35)
$[\hat{\mathbf{a}}^{(J)},\hat{\mathbf{b}}^{(J)}]^T$, $[\hat{\mathbf{a}}_k^{(J)},\hat{\mathbf{b}}_k^{(J)}]^T$ normalized eigenvector $[\mathbf{a}^{(J)},\mathbf{b}^{(J)}]^T$
c, c_J internal length scale parameter [length]^2
$\mathbf{C}_{1k}$, $\mathbf{C}_{2k}$ matrices appearing in the expression for the near-tip displacement jump across the crack $\Delta\mathbf{u}$
$\mathbf{C}'_{1k}$, $\mathbf{C}'_{2k}$ matrices appearing in the expression for the derivative of the near-tip displacement jump across the crack $\partial_y\Delta\mathbf{u}$
$C_{1\ldots4}$ operators of the boundary conditions on traction-free crack faces
$C_{1\ldots4}^{+}$ operators of the boundary conditions on traction-free crack faces of the adjoint problem
E_J Young modulus
$\mathbf{e}_r$, $\mathbf{e}_\varphi$ polar orthonormal base vectors
$f_{kl}^{(J)}$, $g_{kl}^{(J)}$ angular functions of the classical elasticity stress tensor component
$\mathbf{F}^{(J)}$ matrix of expressions appearing in the boundary conditions on crack faces
$\mathbf{F}$ eigenvalue problem matrix composed of the matrices $\mathbf{F}^{(J)}$ and $\mathbf{G}^{(J)}$
G^{int} total interfacial energy release rate
G_1^{int}, G_2^{int} Mode I and Mode II energy release rates
$\mathbf{G}^{(J)}$ matrix of expressions appearing in the continuity conditions at the bonded part of the interface
H_k, $\widetilde{H}_k$ complex amplitude factor, modified complex amplitude factor
$\mathcal{H}^h$,$\mathcal{H}$ path independent integrals appearing in the formula for the evaluation of the complex amplitude factor H_k
$I_{1\ldots8}$ special integrals appearing in the ERR expressions
$J = I,II$ material indices
k general index or index of characteristic exponent of the asymptotic field p_k, where $k = 1,2$
$\varkappa_k^{(J)}$ constants appearing as elements of the matrices $\mathbf{S}_{1k}$, $\mathbf{S}_{2k}$, $\mathbf{C}_{1k}$ and $\mathbf{C}_{2k}$
$K = K_1 + iK_2$ classical elasticity complex stress intensity factor
$\mathbf{K}_1$, $\mathbf{K}_{2\ldots5}(p)$ matrices of the differential operator $\mathcal{L}(p)$
$\mathbf{K}_1^{+}$, $\mathbf{K}_{2\ldots5}^{+}(p)$ matrices of the adjoint differential operator $\mathcal{L}^{+}(p)$
l reference length
$\mathcal{L}(p)$ ordinary fourth-order differential operator
$\mathcal{L}^{+}(p)$ adjoint differential operator
$\mathbf{m}^{(J)}$,$m_{jkl}^{(J)}$ dipolar stress tensor and its jkl component
$\mathbf{m}_r^{(J)}$, $m_{rkl}^{(J)}$ vector of the asymptotic dipolar stress tensor components rkl
$\mathbf{m}_\varphi^{(J)}$, $m_{\varphi kl}^{(J)}$, $\mathbf{m}_y^{(J)}$, $m_{ykl}^{(J)}$ vector of the asymptotic dipolar stress tensor components φkl and ykl corresponding to $\varphi \neq 0$ and $\varphi = 0$, respectively
$\underset{\sim}{\mathbf{m}}_r^{(J)}$, $\underset{\sim}{m}_{rij}^{(J)}$ angular functions of the asymptotic dipolar stress vector $\mathbf{m}_r$
$\underset{\sim}{\mathbf{m}}_\varphi^{(J)}$, $\underset{\sim}{m}_{\varphi ij}^{(J)}$ angular functions of the asymptotic dipolar stress vector $\mathbf{m}_\varphi$
$\underset{\sim}{m}_{ykl}^{(J)}$ complex-valued components appearing in the expression for the asymptotic dipolar stress vector $\mathbf{m}_y$ on the bonded part of the interface
$\mathbf{m}_r^{(J)*}$, $m_{rkl}^{(J)*}$ complementary dipolar stress vector $\mathbf{m}_r$ and its components
$\mathbf{m}_\varphi^{(J)*}$,$m_{\varphi kl}^{(J)*}$ complementary dipolar stress vector $\mathbf{m}_\varphi$ and its components
$\mathbf{M}_r^{(J)}$,$\mathbf{N}_r^{(J)}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses $\mathbf{m}_r$
$\mathbf{M}_\varphi^{(J)}$,$\mathbf{N}_\varphi^{(J)}$, $\mathbf{M}_y^{(J)}$, $\mathbf{N}_y^{(J)}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses $\mathbf{m}_\varphi$ and $\mathbf{m}_y$, respectively
$\mathbf{M}_1^{(J)}$,$\mathbf{M}_2^{(J)}$ submatrices of the matrix $\mathbf{M}_\varphi^{(J)}$
$\mathbf{N}_1^{(J)}$,$\mathbf{N}_2^{(J)}$ submatrices of the matrix $\mathbf{N}_\varphi^{(J)}$
$\mathbf{n}$ outer normal unit vector
$O_{1\ldots4}$ operators of the transmission conditions at the bonded interface
$O_{1\ldots4}^{+}$ operators of the transmission conditions at the bonded interface of the adjoint problem
p, p_k characteristic exponent of the asymptotic field