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Asymptotic solution for interface crack between two materials governed by dipolar gradient elasticity

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ARTICLE INFO	A B S T R A C T		
Keywords:	An asymptotic solution for interface crack between mismatched materials that obey a special		
Dipolar gradient elasticity	form of linear isotropic strain gradient elasticity under conditions of plane strain is developed. It		
Interface crack Asymptotics	is shown that the asymptotic solution depends in a complicated manner both on a mismatch of		
	elastic properties and a mismatch of the internal length scale parameter. Numerical analysis		
	shows that the mismatch of elastic moduli and the gradient coefficient c respectively lift the		
	degeneracy of the exponent p that is characteristic for a crack in homogeneous material. The total		
	energy release rate G^{int} due to a crack extension along the interface is derived generalizing the		
	classical virtual crack closure method. The reciprocal work contour integral method for the		
	evaluation of amplitude factors in the asymptotic expansion is extended to interface crack in the		

linear isotropic strain gradient elasticity.

1. Introduction

The paper deals with the determination of the asymptotic displacement, strain and stress fields in the vicinity of the interface crack between two mismatched materials governed by dipolar gradient elasticity which by introducing intrinsic length scale allows predicting the scale effects observed experimentally. The gradient elasticity and, in general, higher order continua theories have recently attracted the attention of researchers in various disciplines due to the ability of the higher-order or nonlocal terms to model phenomena which cannot be described by classical elasticity since it does not include an internal length scale in its constitutive structure. Among these phenomena belong for example the occurrence of size effects (i.e. the dependence of strength or other macroscopic properties on specimen size), as well as fracture and/or interface processes where the detailed "non-singular" or "continuous" distribution of stress and strain fields near the crack tip or the interface is of prime importance. Currently an increasing effort to capture effects of microstructure in the framework of generalized continuum theories is driven by emerging of new materials and devices with the characteristic size lengths comparable to the lengths of material microstructure. The intention is to extend the range of applicability of the continuum concept and thus to contribute to bridging the gap between classical continuum theories and atomic-lattice theories. A major reason of this effort is that molecular dynamics and/or quantum mechanics based simulations require tremendous computation resources. A general higher-order elastic theory was proposed by Mindlin [1]. For practical purposes, Mindlin also formulated three simplified versions of the general isotropic theory, utilizing only two material and five internal lengthscale constants in the final constitutive relation rather than 18 used in Mindlin's initial model. According to the inherent form of the strain energy density these three versions are specified as Form I, II and III. In the case of the Form I is the strain energy density a quadratic form of the classical strains and the second gradient of displacement. In the case of the Form II is the strain energy density a quadratic form of the classical strains and the gradient of strains and in the case of the Form III is the strain energy density a quadratic

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$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Nomenclature		$\mathbf{e}_r, \mathbf{e}_{\varphi}$	polar orthonormal base vectors	
Mathematical symbols Mathematical symbols tensor product scalar product scalar product D = n.V inversion the contour with the normal $L = (-1)^{1/2}$ complex unit V = V = n.D surface gradient operator on the contour with the normal unit vector n V = V = n.D surface gradient operator on the contour with the normal unit vector n V = V = n.D surface gradient operator V = V = 0 surface gradient operator V = 0			$f_{kl}^{(J)}, g_{kl}^{(J)}$	angular functions of the classical elasticity stress	
Solutionmatrix of expression appearing in the boundary conditions on crack facesSolutionensor productD = n·V, normal derivative on the contour with the normal unit vector neigenvalue problem matrix composed of the ma- trices F ⁽ⁱ⁾ and G ^(j) Q ⁱ Laplacian operatorG ⁽ⁱⁱⁱ⁾ Q ⁱ V = To surface gradient operator on the contour with the normal unit vector nG ⁽ⁱⁱⁱ⁾ Q ⁱ V = N-DSurface gradient operator on the contour with the normal unit vector nIunit dyadic tensorH _i , fiIunit dyadic tensorH _i , fi(I)jump of a function facross the interfaceH ⁽ⁱ⁾ , fi(I)gamma functionJ = I,IISiand carsin coordinatesgeneral index or index of characteristic exponent of the asymptotic field p _i , where k = 1,2(I)gamma functionspecial integrals appearing in the ERR expressions J = I,II(I)gamma functionkspecial integrals appearing as elements of the matrices(I)Gamma functionksepcial integrals appearing in the expression J = I,II(I)gamma part of the singularity exponent in the classical elasticitykk(I)Lamé constantkk(I)Lamé constantkk(I)kterses component in the classical elasticitymiterial operator(I)kterses vector r(i)k(I)kkcomponent id(I)kfinite crack extension for the asymptotic dipolar stress vector m, an	Mathematical symbols		-(1)	tensor component	
	-	1	$\mathbf{F}^{(j)}$	matrix of expressions appearing in the boundary	
$ f_{1} = n \cdot \nabla \text{ morinal derivative on the contour with the normal unit vector n f_{2} = partal derivative with respect to the coordinate k or arrange of the interface or on the interface or interface of the interface or interface of the interface of the interface or interface of the interface or on the interface of the interface of the interface of the interface or interface of the anyunptotic dipolar stress vector material periator of the asymptotic dipolar stress vector material periator of the asymptotic dipolar stress vector material periator interface of the anyunptotic dipolar stress vector material periator interface of the anyunptotic dipolar stress vector material periator of the asymptotic dipolar stress were material periator of the asymptotic dipolar stress material interface of the anyunptotic dipolar stress material interface of the anyunptotic dipolar stress material inthe components in the corresta material perinter of the a$	\otimes	tensor product	-	conditions on crack faces	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	D	scalar product	F	eigenvalue problem matrix composed of the matrices $\mathbf{E}^{(J)}$ and $\mathbf{C}^{(J)}$	
$ \begin{aligned} & \int_{\mathbb{T}} \frac{\partial f_{i}}{\partial f_{i}} & \int_{\mathbb{T}}^{d_{i}} Mode I and Mode II energy release rates \\ & \forall gradient operator \\ & \forall = V = D. surface gradient operator on the contour with the normal unit vector n is the contact of the interface matrix of expressions appearing in the continuity conditions at the bonded part of the interface H_k, \tilde{H}_k. Complex amplitude factor, modified complex amplitude factor H^k, \tilde{H}_k, \tilde{H}_$	$D = \mathbf{n} \cdot \mathbf{v}$	normal derivative on the contour with the normal	Cint	total interfacial energy release rate	
$\begin{array}{llllllllllllllllllllllllllllllllllll$	a	unit vector n partial derivative with respect to the coordinate k	G ^{int} G ^{int}	Mode I and Mode II energy release rates	
$ \begin{aligned} \nabla^2 & Laplacian operator \\ \nabla^2 & Laplacian operator \\ \nabla^2 & V = N-D Surface gradient operator on the contour with the normal unit vector n the normal unit dyadic tensor (x, φ), (x, ψ) polar and Cartesian coordinates (x, ψ), (x, ψ) porterial distribution (x, ψ), (x, ψ) polar and (x, ψ) ($	∇_k	gradient operator	$\mathbf{G}^{(J)}_{I}$	matrix of expressions appearing in the continuity	
$\begin{aligned} & f_{1} & \nabla_{-D} & \text{surface gradient operator on the contour with the normal unit vector n \\ & f_{2} & \nabla_{-D} & \text{surface gradient operator on the contour with the normal unit vector n \\ & f_{2} & \nabla_{-D} & \text{surface gradient operator on the contour with the normal unit vector n \\ & f_{2} & \nabla_{-D} & surface gradient operator on the contour with the normal unit dyadic tensor f anti-diagonal f anti-$	∇^2	Lanlacian operator		conditions at the bonded part of the interface	
$ \begin{aligned} \mathbf{v} = \mathbf{v} - \mathbf{u} \\ \mathbf{u} \text{ it dyadic tensor } \\ \mathbf{f} \\ f$	\hat{s}	D surface and instances on the contour with	H_k, \widetilde{H}_k	complex amplitude factor, modified complex am-	
$\begin{array}{lll} \begin{aligned} \mu = (-1)^{1/2} & \operatorname{complex} unit \\ \mu = ($	v = v - nD surface gradient operator on the contour with			plitude factor	
$ \begin{aligned} I & unit dyadic tensor \\ \hline I & anti-diagonal unit dyadic tensor \\ \hline I & costant \\ \hline P & closed contour encircling the crack tip \\ e & imaginary part of the singularity exponent in the classical elasticity \\ e^{t/0}, e^{(1)}_{ij} & strain tensor and its kl - component \\ \phi^{(1)}_{ij}, f^{(2)}_{ij} & kl - stress tensor and its kl - component \\ \phi^{(1)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - component \\ \phi^{(1)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - component \\ \phi^{(1)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - component \\ \phi^{(1)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - component \\ \phi^{(2)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - component \\ \phi^{(2)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - component \\ \phi^{(2)}_{ij}, f^{(2)}_{ij} & nonpolar stress tensor and its kl - components kl \\ A_{ij}, B_{ij}^{(2)}_{ij} & nonpolar stress ten$	$i = (-1)^{1}$	$\frac{1}{2}$ complex unit	$\mathcal{H}^h, \mathcal{H}$	path independent integrals appearing in the for-	
$ f_{i} anti-diagonal unit dyadic tensor (r.φ), (xy) polar and Cartesian coordinates g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices general index or index of characteristic exponent of the asymptotic field p_{k}, where k = 1, 2 g_{i} = I, II material indices g_{i} = I, II material indices general index or index of the matrices g_{i} = I, i \in I, II material indices K = K_{i} + iK_{i} = (assice) field asticity or milex stress intensity factor K_{i} = K_{i} = i, II material indices K = K_{i} + iK_{i} = (assice) field asticity or milex expression for the asymptotic dipolar stress tess intensity f_{i} = 0, respectively g_{i} = 0, respec$	l = (-1)	unit dvadic tensor		mula for the evaluation of the complex amplitude	
$\begin{array}{lll} I_{j} & \text{spin} (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (x_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi_{j}), (r, \varphi_{j}) & \text{polar and Cartesian coordinates} \\ (r, \varphi_{j}), (r, \varphi$	$\widehat{\mathbf{I}}$ unit dyadic tensor			factor H_k	
$ \begin{aligned} J = I,II & \text{material indices} \\ general indices \\ general indices \\ general indices \\ general indices \\ general index or index of characteristic exponent of the asymptotic field p_k, where k = 1,2 x_k^{(J)} & \text{constant appearing as elements of the matrices } \\ S_{1k}, S_{2k}, C_{1k}, C_{1k}, and C_{2k} \\ S_{1k}, S_{2k}, C_{1k}, and S_{2k} \\ S_{1k}, S_{2k}, C_{1k}, an$	$(r \sigma) (r)$	y) polar and Cartesian coordinates	I_{18}	special integrals appearing in the ERR expressions	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	(1, 4), (A,	real and imaginary part of a complex expression	J = I, II	material indices	
$ \begin{aligned} f_{1} & \text{Gramma function} \\ f_{1} & \text{Gamma function} \\ f_{1} & \text{Gamma function} \\ f_{2} & \text{Grawna function} \\ f_{2} & \text{Grawna function} \\ f_{3} & \text{Dundurs parameter} \\ f_{1} & \text{closed contour encircling the crack tip} \\ \varepsilon & \text{imaginary part of the singularity exponent in the classical elasticity} \\ f_{1} & \text{closed contour encircling the crack tip} \\ \varepsilon & \text{imaginary part of the singularity exponent in the classical elasticity} \\ f_{2} & \text{Choson's constant} \\ f_{2} & \text{Lamé constant} \\ f_{2} & f_{2} & \text{Lamé constant} \\ f_{2} & f_{2} & \text{Lamé constant} \\ f_{2} & \text{Lamé constant} \\ f_{2} & f_{2} & \text{Lamé constant} \\ f_{2} & \text{Lamé constant} \\ f_{2} & \text{Lamé constant} \\ f_{2} & f_{2} & f_{2} & \text{Lamé constant} \\ f_{2} & f_{2} & f_{2} & \text{Lamé constant} \\ f_{2} & f_{2} & f_{2} & Lamé cons$	<i>J</i> (, <i>J</i> ∏ <i>f</i>]]	iump of a function f across the interface	k	general index or index of characteristic exponent	
$ \begin{aligned} Greek symbols \\ Greek symbols \\ \beta \\ Dundurs parameter \\ \Gamma \\ classical elasticity exponent in the classical elasticity complex stress intensity factor \\ factor \\$	Г(.)	Gamma function		of the asymptotic field p_k , where $k = 1,2$	
Greek symbols $S_{kk} = S_{kk} = C_{kk} = C_{kk} = C_{kk}$ β Dundurs parameter Γ closed contour encircling the crack tip ε imaginary part of the singularity exponent in the $classical elasticity\varepsilon\varepsilonimaginary part of the singularity exponent in the(f), \xi_{kl}^{(1)}strain tensor and its kl component\kappa_{j}Kolosov's constant\mu_{j}Lamé constant\mu_{j}shear modulus\psi_{j}Poisson's ratio\sigma_{kl}^{(1)}kl -stress component in the classical elasticity\tau^{(1)}, \tau_{kl}^{(1)}monopolar stress tensor and its kl -component inthe gradient elasticity\tau^{(1)}, \tau_{kl}^{(1)}monopolar stress tensor and its kl -component inthe gradient elasticity\tau^{(1)}, \tau_{kl}^{(1)}monopolar stress tensor and its kl -components rklm_{j}^{(1)}, m_{kl}^{(1)}wetter of the asymptotic dipolar stress\tau^{(1)}, \tau_{kl}^{(1)}monopolar stress tensor and its kl -components rklm_{j}^{(1)}, m_{kl}^{(1)}monopolar stress tensor \tau^{(1)}\psi_{j}phase angle\psi_{j}, \psi_{k}phase angleLatin symbolsfinite crack extensionA_{m}^{(1)}, n_{j}^{(1)}, n_{kl}^{(1)}A_{m}^{(1)}, n_{j}^{(1)}A_{m}^{(1)}, n_{j}^{(1)}, n_{kl}^{(1)}A_{m}^{(1)}, n_{j}^{(1)}, n_{kl}^{(1)}A_{m}^{(1)}, n_{j}^{(1)}, n_{kl}^{(1)}\mu_{j}^{(1)}, n_{kl}^{(1)}\mu_{j}^{(1)}, n_{kl}^{(1)}\mu_{j}^{(1)}, n_{kl}^{(1)}\mu_{j}^{(1)}, n_{kl}^{(1)}<$			$\kappa_k^{(J)}$	constants appearing as elements of the matrices	
$\beta \qquad \text{Dundurs parameter} Γ closed contour encircling the crack tip ε imaginary part of the singularity exponent in the classical elasticity \tau^{(f)}, \tau_{kl}^{(f)} monopolar stress tensor and its kl component inthe gradient elasticityz^{(f)} magular functions of the asymptotic monopolarstress vector \tau^{(f)} with espect to \tau^{(f)} by\psi_{k} phase angleLatin symbolsLatin symbolsLatin$	Greek syı	nbols		$\mathbf{S}_{1k}, \mathbf{S}_{2k}, \mathbf{C}_{1k} \text{ and } \mathbf{C}_{2k}$	
$ \begin{array}{lll} \beta & \text{Dundurs parameter} \\ \Gamma & \text{closed contour encircling the crack tip} \\ \varepsilon & \text{imaginary part of the singularity exponent in the classical elasticity} \\ \varepsilon^{(J)}, \varepsilon^{(J)}_{kl}, \\ \varepsilon^{(J)}, \varepsilon^{(J)}_{kl} & \text{strain tensor and its kl component} \\ k_{f} & \text{Kolosov's constant} \\ \lambda_{f} & \text{Lamé constant} \\ \lambda_{f} & \text{Lamé constant} \\ \lambda_{f} & \text{Baer modulus} \\ \psi_{f} & \text{Poisson's ratio} \\ \sigma^{(J)}_{kl}, \\ kl - stress component in the classical elasticity \\ \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its kl -component in the gradient elasticity \\ \varepsilon^{(J)}, \varepsilon^{(J)}_{kl} & \text{angular functions of the asymptotic monopolar stress vector \tau^{(J)} \psi_{kk} & \text{phase angle} \\ \lambda_{f} & \text{phase angle} \\ \lambda_{f} & \text{phase angle} \\ \lambda_{f} & \beta_{f} & \beta_{f} & \text{elements of the eigenvector [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^{T} \\ A^{(J)}, \mathbf{b}^{(J)}_{kl} & \text{derivative of the elements of the matrices \mathbf{A}^{(J)} and \mathbf{b}^{(J)} & \mathbf{b}^{(J)} \\ \mathbf{a}^{(J)}, \mathbf{b}^{(J)}_{kl} & \mathbf{b}^{(J)}_{kl} & \text{derivative of the elements of the asymptotic displacements \mathbf{u}^{(J)} \\ \mathbf{a}^{(J)}, \mathbf{b}^{(J)}_{kl} & \mathbf{b}^{(J)}_{kl} &$	·		$K = K_1 +$	iK_2 classical elasticity complex stress intensity	
$ \begin{aligned} \Gamma & closed contour encircling the crack tip imaginary part of the singularity exponent in the classical elasticity exponent in the classical elasticity follow, for the symptotic dipolar stress tensor and its kl component in the classical elasticity for the symptotic monopolar stress tensor and its kl -component in the classical elasticity for the gradient elasticity for the gradient elasticity for the symptotic monopolar stress tensor and its kl -component in the classical elasticity for the gradient elasticity for the gradient elasticity for the symptotic monopolar stress tensor and its kl -component in the classical elasticity for the gradient elasticity for the gradient elasticity for the symptotic for the asymptotic monopolar stress vector τ(I) for the asymptotic dipolar stress tensor for the asymptotic dipolar stress tensor for the asymptotic dipolar stress vector πk for the asymptotic dipolar stress tensor for the asymptotic dipolar stress vector mk for the asymptotic dipolar stress tensor for the asymptotic dipolar stress vector mk and the for the asymptotic dipolar stress vector mk and the for the asymptotic dipolar stresses mk for the asymptotic for the asymptotic for the asymptotic dipolar stress vector mk and the for the asymptotic dipolar stresses mk and mk, respectively M(1), M(1) with matrices of the angular functions in the expression for the asymptotic dipolar stresses mk and mk, respectively M(2), M(2) submatrices of the matrix M(2) for the asymptotic dipolar stresses mk and mk, respectively M(2) with matrices of the asymptotic dipolar stresses mk and mk, respectively M(2) with matrices of the asymptotic dipolar stresses mk and mk, respectively M(2) with matrices of the asymptotic dipolar stresses mk and mk, respectively M(2) with matrices of the asymptotic dipolar stresses mk and mk, respectively M(2) with matrices of the asymptotic dipolar stresses mk and mk, respectively M(2) with matrices of the asymptotic dipolar str$	β	Dundurs parameter		factor	
$ \begin{aligned} \varepsilon & \text{imaginary part of the singularity exponent in the classical elasticity \\ \varepsilon^{(I)}, \varepsilon^{(J)}_{ij}, \\ \varepsilon^{(J)}_{ij}, \varepsilon^{(J)}_{ij} & \text{train tensor and its kl component \\ k_{f} & \text{Kolosov's constant} \\ \lambda_{f} & \text{Lamé constant} \\ \lambda_{f} & \text{Lamé constant} \\ \lambda_{f} & \text{Lamé constant} \\ \lambda_{f} & \text{Iamé constant} \\ \lambda_{f} & \text{Iamé constant} \\ \lambda_{f} & \text{Iamé constant} \\ \lambda_{f} & \text{Istress remodulus} \\ \psi_{f} & \text{Poisson's ratio} \\ \varepsilon^{(J)}, \varepsilon^{(J)}_{kl}, \\ \varepsilon^{(J)}, \varepsilon^{(J)}_{kl} & \text{monopolar stress tensor and its kl -component in the classical elasticity \\ \tau^{(J)}, \varepsilon^{(J)}_{kl} & \text{angular functions of the asymptotic monopolar stress vector $\tau^{(J)}$ were vector μ_{p} way were the angular functions of the asymptotic dipolar stress vector μ_{p} were wattrees of the eigenvector $[a^{(J)}, b^{(J)}]^{T}$ and $\mu_{p}^{(J)}, $a^{(J)}_{kl}, $b^{(J)}_{kl} \ derivative of the elements of the matrices $A^{(J)}$ and $\mu_{p}^{(J)}, $a^{(J)}_{kl}, $b^{(J)}_{kl} \ derivative of the elements of the matrices $A^{(J)}$ and $\mu_{p}^{(J)}, $a^{(J)}_{kl}, $b^{(J)}_{kl}, $\mu_{p}^{(J)}$ derivative of the elements of the matrices $A^{(J)}$ and $\mu_{p}^{(J)}, $a^{(J)}_{kl}, $b^{(J)}_{kl}, $\mu_{p}^{(J)} \ derivative of the elements of the matrices $A^{(J)}$ and $\mu_{p}^{(J)}, $a^{(J)}_{kl}, $b^{(J)}_{kl}, $\mu_{p}^{(J)}$ derivative of the elements of the matrices $A^{(J)}$ and $\mu_{p}^{(J)}, $a^{(J)}_{kl}, $b^{(J)}_{kl}, $\mu_{p}^{(J)}$ more and and $\mu_{p}^{(J)}$ and $\mu_{p}^{(J)}$$	Г	closed contour encircling the crack tip	$K_1, K_{25}($	p) matrices of the differential operator $\mathcal{L}(p)$	
classical elasticity $\epsilon^{(J)}, \epsilon^{(J)}_{kl}$, strain tensor and its kl component k_{f} Kolosov's constant μ_{f} Shear modulus ψ_{f} Poisson's ratio $\sigma^{(J)}_{kl}$, kl -stress component in the classical elasticity $\tau^{(J)}, \tau^{(J)}_{kl}$, kl-stress component in the classical elasticity $\tau^{(J)}, \tau^{(J)}_{kl}$, kl-stress component in the classical elasticity $\tau^{(J)}, \tau^{(J)}_{kl}$, monopolar stress tensor and its kl -component in the gradient elasticity t , ψ_{k} phase angle ψ, ψ_{k} phase angle $kl^{(J)}, \mathbf{b}^{(J)}_{kl}$, $\mathbf{b}^{(J)}_{l-k}$, elements of the asymptotic monopolar $\mathbf{A}^{(J)}_{k-k}, \mathbf{B}^{(J)}_{k-k}$, elements of the eigenvector $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^{T}$ $\mathbf{A}^{(J)}_{m}, \mathbf{B}^{(J)}_{m}$ derivative of the elements of the matrices $\mathbf{A}^{(J)}$ and $\mathbf{B}^{(J)}$ with respect to φ $\mathbf{A}^{(J)}_{m}, \mathbf{B}^{(J)}_{m}$ derivative of the elements of the matrices $\mathbf{A}^{(J)}$ and $\mathbf{B}^{(J)}, \mathbf{b}^{(J)}_{m}]^{T}$ internal length scale parameter [length] ² $\mathbf{C}_{1k}, \mathbf{C}_{2k}$ matrices appearing in the expression for the asymptotic dipolar stresses \mathbf{m}_{τ} $\mathbf{M}^{(J)}_{m}, \mathbf{N}^{(J)}_{m}, \mathbf{N}^{(J)}_{m}, \mathbf{N}^{(J)}_{m}$ matrices of the angular functions in the expression for the asymptotic dipolar stress vector \mathbf{m}_{τ} and its $\mathbf{C}^{(J)}_{m}, \mathbf{C}^{(J)}_{m}$ internal length scale parameter [length] ² $\mathbf{C}_{1k}, \mathbf{C}_{2k}$ matrices appearing in the expression for the near- ity displacement jump across the crack $\Delta \mathbf{u}$ \mathbf{C}_{m} $\mathbf{C}_{m}^{(J)}, \mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}$ submatrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{τ} $\mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}$ submatrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{τ} $\mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{τ} $\mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}, \mathbf{M}^{(J)}_{m}$ matrices of the angular func	ε	imaginary part of the singularity exponent in the	$\mathbf{K}_{1}, \mathbf{K}_{25}$	(p) matrices of the adjoint differential operator	
$ \begin{aligned} \varepsilon^{(J)}, \varepsilon^{(J)}_{kl}, \\ \xi^{(J)}_{l}, \xi^{(J)}_{kl} \\ Lamé constant \\ \lambda_{j} \\ Lamé constant \\ \lambda_{j$		classical elasticity	$\mathcal{L}^{\gamma}(p)$	reference length	
$ \begin{aligned} & \chi_{l} & \text{Kolosov's constant} \\ & \lambda_{l} & \text{Lamé constant} \\ & \mu_{j} & \text{shear modulus} \\ & \mu_{j} & \text{stress component in the classical elasticity} \\ & \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ & \tau^{(J)}, \pi^{(J)}_{kl} & \text{monopolar stress vector } \tau^{(J)} \\ & \text{monopolar stress vector } \pi^{(J)} \\ & monop$	$\boldsymbol{\varepsilon}^{(J)}, \boldsymbol{\varepsilon}^{(J)}_{kl}$	strain tensor and its kl component	$\int (n)$	ordinary fourth order differential operator	
$\begin{array}{llllllllllllllllllllllllllllllllllll$	κ_J	Kolosov's constant	$\mathcal{L}(p)$ $f^+(n)$	adjoint differential operator	
$ \begin{array}{ll} \mu_{j} & \text{shear modulus} \\ \mu_{j} & \text{poisson's ratio} \\ \nabla_{kl}^{(j)} & kl - \text{stress component in the classical elasticity} \\ \tau^{(j)}, \tau^{(j)}_{kl} & \text{monopolar stress tensor and its kl -component in the gradient elasticity} \\ \tau^{(j)}, \tau^{(j)}_{kl} & \text{monopolar stress tensor and its kl -component in the gradient elasticity} \\ \tau^{(j)} & \text{angular functions of the asymptotic monopolar stress vector } \tau^{(j)} & \mu_{j}^{(j)} & \mu_{j}^{(j)}, m_{j}^{(j)}, m_{j}^{(j)} & m_{j}^{(j)}, m_{j}^{(j)} & m_$	λ_J	Lamé constant	$\mathcal{L}(p)$ $\mathbf{m}^{(J)} \mathbf{m}^{(J)}$	dipolar stress tensor and its <i>ikl</i> component	
<i>p</i> _{<i>f</i>} Poisson's ratio <i>σ</i> _{<i>kl</i>} ^(f) <i>kl</i> -stress component in the classical elasticity <i>τ</i> ^(f) , <i>τ</i> _{<i>kl</i>} ^(f) monopolar stress tensor and its <i>kl</i> -component in the gradient elasticity <i>τ</i> ^(f) angular functions of the asymptotic monopolar stress vector <i>τ</i> ^(f) <i>φ</i> , <i>ψ</i> _{<i>k</i>} phase angle <i>μ</i> , <i>ψ</i> _{<i>k</i>} phase angle <i>Latin symbols</i> <i>Latin symbolic displacements u^(f) <i>A</i>^(f), <i>B</i>^(f) <i>B</i>^(f), <i>B</i>^(f) <i>B</i>^(f), <i>B</i>^(f) <i>B</i>^(f), <i>B</i>^(f) <i>B</i>^(f), <i>B</i>^(f), <i>B</i>^(f) <i>B</i>^(f), <i>B</i>^{(f}</i>	μ_J	shear modulus	(J) = (J)) voctor of the computation display stress tensor	
$\begin{aligned} \tau^{(J)}, \tau^{(J)}_{kl}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ \\ \tau^{(J)}, \tau^{(J)}_{kl} & \text{monopolar stress tensor and its } kl - component in the gradient elasticity \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	ν_J	Poisson's ratio	\mathbf{m}_{r} , m_{rk}	components rkl	
$ \begin{aligned} & \tau_{0}^{(J)}, \tau_{kl}^{(J)} & \text{monopolar stress tensor and its kl -component in the gradient elasticity } \\ & \tau_{0}^{(J)} & \text{angular functions of the asymptotic monopolar stress vector \tau^{(J)} angular functions of the asymptotic monopolar stress vector \tau^{(J)} with phase angle \psi, \psi_k phase angle \psi, \psi, \psi_k phase angle \psi, \psi_k phase angle \psi, \psi$	$\sigma_{kl}^{(s)}$	<i>kl</i> -stress component in the classical elasticity	$\mathbf{m}^{(J)} \mathbf{m}^{(J)}$	$\vec{\mathbf{m}}^{(J)} = \vec{\mathbf{m}}^{(J)} \mathbf{m}^{(J)}$ vector of the asymptotic dipolar stress	
the gradient elasticity $z^{(I)}$ angular functions of the asymptotic monopolar stress vector $\tau^{(I)}$ ψ, ψ_k phase angle Latin symbols Latin symbols Δa finite crack extension $A_{14}^{(I)}, B_{14}^{(I)}$ elements of the eigenvector $[\mathbf{a}^{(I)}, \mathbf{b}^{(I)}]^T$ $\mathbf{A}^{(J)}, \mathbf{B}^{(J)}$ matrices of the angular functions in the expression for the asymptotic displacements $\mathbf{u}^{(I)}$ $\mathbf{a}^{(J)}, \mathbf{b}^{(J)}$ angular functions of the asymptotic dipolar stress vector \mathbf{m}_{φ} $\mathbf{m}^{(J)}_{\varphi, \psi}$ angular functions of the asymptotic dipolar stress vector $\mathbf{m}_{\varphi}^{(J)}, \mathbf{m}^{(J)}_{\varphi(I)}$ angular functions of the asymptotic dipolar stress vector \mathbf{m}_{φ} $\mathbf{m}^{(J)}_{\psi, \psi}$ complementary dipolar stress vector \mathbf{m}_{r} and its components $\mathbf{m}^{(J)}_{\varphi}, \mathbf{m}^{(J)}_{\psi(I)}$ complementary dipolar stress vector \mathbf{m}_{φ} and its components $\mathbf{m}^{(J)}_{\varphi}, \mathbf{m}^{(J)}_{\psi(I)}$ complementary dipolar stress vector \mathbf{m}_{φ} and its components $\mathbf{m}^{(J)}_{\varphi}, \mathbf{m}^{(J)}_{\psi(I)}$ matrices of the angular functions in the expression for the asymptotic displacement stress in for the near- tip displacement jump across the crack Δu $\mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{r} $\mathbf{M}^{(J)}_{\varphi}, \mathbf{N}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{φ} $\mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{φ} $\mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}$ matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{φ} and \mathbf{m}_{y} , respectively $\mathbf{M}^{(J)}_{\varphi}, \mathbf{M}^{(J)}_{\varphi}$ submatrices of the matrix $\mathbf{M}^{(J)}_{\varphi}$	$\tau^{(J)}, \tau^{(J)}_{kl}$	monopolar stress tensor and its kl -component in	$m_{\varphi}, m_{\varphi k}$	tensor components αkl and νkl corresponding to	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	-(I)	the gradient elasticity	$\omega \neq 0$ an	$d \varphi = 0$, respectively	
$\begin{aligned} & \psi, \psi_k \\ & phase angle \\ & \mathcal{Latin symbols} \\ & \Delta a \\ & finite crack extension \\ & A_{1,\dots,4}^{(J)}, B_{1,\dots,4}^{(J)} \\ & elements of the eigenvector [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T \\ & \mathbf{A}^{(J)}, \mathbf{B}^{(J)}_{\dots} \\ & \text{matrices of the angular functions in the expression for the asymptotic displacements \mathbf{u}^{(J)} \\ & \mathbf{A}^{(J)}, \mathbf{B}^{(J)}_{\dots} \\ & \mathbf{B}^{(J)} \\ & \mathbf{b}^{(J)}_{\dots} \\ & \mathbf{b}^{(J$	$\overset{1}{\sim}$	stress vector $\sigma^{(J)}$	$\mathbf{m}^{(J)}, m^{(J)}$	⁽⁾ angular functions of the asymptotic dipolar stress	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	1 /2 1 /2	phase angle	$\sim r' \sim rij$	vector \mathbf{m}_r	
<i>Latin symbols</i> <i>Latin symbols</i> Δa finite crack extension $A_{1\dots4}^{(J)}, B_{1\dots4}^{(J)}$ elements of the eigenvector $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$ $\mathbf{A}_{r}^{(J)}, \mathbf{B}_{r}^{(J)}$ matrices of the angular functions in the expression for the asymptotic displacements $\mathbf{u}^{(J)}$ $\mathbf{A}_{\varphi}^{(J)}, \mathbf{B}_{\varphi}^{(J)}$ derivative of the elements of the matrices $\mathbf{A}^{(J)}$ and $\mathbf{B}^{(J)}$ with respect to φ $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$ eigenvector in the eigenvalue problem (35) $[\mathbf{\hat{a}}^{(J)}, \mathbf{\hat{b}}^{(J)}]^T$, $[\mathbf{\hat{a}}_{k}^{(J)}, \mathbf{\hat{b}}_{k}^{(J)}]^T$ normalized eigenvector $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$ c, c_f internal length scale parameter [length] ² $\mathbf{C}_{1k}, \mathbf{C}_{2k}$ matrices appearing in the expression for the near- tip displacement jump across the crack $\Delta \mathbf{u}$ $\mathbf{C}_{1k}^{(J)}, \mathbf{C}_{2k}^{(J)}$ matrices appearing in the expression for the deri-	φ, φ_k	pilase angle	$\mathbf{m}_{n}^{(J)}, m_{n}^{(J)}$	⁽⁾ angular functions of the asymptotic dipolar stress	
$\begin{aligned} & \Delta a & \text{finite crack extension} \\ & A^{(J)}_{1,\dots,4}, B^{(J)}_{1,\dots,4} & \text{elements of the eigenvector } [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T \\ & \mathbf{A}^{(J)}, \mathbf{B}^{(J)}_{1,\dots,4} & \text{elements of the angular functions in the expression} \\ & \text{for the asymptotic displacements } \mathbf{u}^{(J)} \\ & \mathbf{A}^{(J)}_{,,,,,,,,,,,,,,,,,,,,}, \mathbf{B}^{(J)}_{,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,$	Latin symbols		$\sim \varphi \sim \varphi_{i}$	vector \mathbf{m}_{φ}	
$\begin{array}{llllllllllllllllllllllllllllllllllll$			$m_{vkl}^{(J)}$	complex-valued components appearing in the ex-	
$ \begin{array}{l} \mathbf{A}_{i,\omega,4}^{(J)}, \mathbf{B}_{i,\omega}^{(J)} & \text{elements of the eigenvector } [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T \\ \mathbf{A}_{\varphi}^{(J)}, \mathbf{B}_{\varphi}^{(J)} & \text{matrices of the angular functions in the expression for the asymptotic displacements \mathbf{u}^{(J)} \\ \mathbf{A}_{\varphi}^{(J)}, \mathbf{B}_{\varphi}^{(J)} & \text{derivative of the elements of the matrices } \mathbf{A}^{(J)} \text{ and } \\ \mathbf{B}^{(J)} \text{ with respect to } \varphi \\ [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T & \text{eigenvector in the eigenvalue problem (35)} \\ [\mathbf{\hat{a}}^{(J)}, \mathbf{\hat{b}}^{(J)}]^T, [\mathbf{\hat{a}}_{k}^{(J)}, \mathbf{\hat{b}}_{k}^{(J)}]^T & \text{normalized eigenvector } [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T \\ c, c_f & \text{internal length scale parameter [length]^2} \\ \mathbf{C}_{1k}, \mathbf{C}_{2k} & \text{matrices appearing in the expression for the nearting the expression for the expression for the expression for the derivative of the asymptotic dipolar stresses \mathbf{m}_{\varphi} \\ \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{y}^{(J)}, \mathbf{N}_{y}^{(J)} & \text{matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{\varphi} \\ \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{y}^{(J)}, \mathbf{N}_{y}^{(J)} & \text{matrices of the angular functions in the expression for the asymptotic dipolar stresses \mathbf{m}_{\varphi} \\ \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{y}^{(J)}, \mathbf{M}_{y}^{(J)} & \text{submatrices of the matrix } \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)} & \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)} & \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)} & \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)} $	Δa	finite crack extension	, • yki	pression for the asymptotic dipolar stress vector	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	$A_{1\dots 4}^{(J)}, B_{1\dots 4}^{(J)}$ elements of the eigenvector $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$			\mathbf{m}_{y} on the bonded part of the interface	
for the asymptotic displacements $\mathbf{u}^{(J)}$ $\mathbf{A}_{,\varphi}^{(J)}, \mathbf{B}_{,\varphi}^{(J)}$ derivative of the elements of the matrices $\mathbf{A}^{(J)}$ and $\mathbf{B}^{(J)}$ with respect to φ $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$, $[\mathbf{\hat{a}}_{k}^{(J)}, \mathbf{\hat{b}}_{k}^{(J)}]^T$ normalized eigenvector $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$ c, c_J internal length scale parameter [length] ² $\mathbf{C}_{1k}, \mathbf{C}_{2k}$ matrices appearing in the expression for the near- tip displacement jump across the crack $\Delta \mathbf{u}$ $\mathbf{C}_{1k}', \mathbf{C}_{k}'$ matrices appearing in the expression for the deri-	$\mathbf{A}^{(J)}$, $\mathbf{B}^{(J)}$ matrices of the angular functions in the expression		$\mathbf{m}_{r}^{(J)*}, m_{r}^{(J)}$	${}^{(J)*}_{kl}$ complementary dipolar stress vector \mathbf{m}_r and its	
$ \begin{aligned} \mathbf{A}_{\varphi}^{(J)}, \mathbf{B}_{\varphi}^{(J)} & \text{derivative of the elements of the matrices } \mathbf{A}^{(J)} \text{ and } \\ \mathbf{B}^{(J)} \text{ with respect to } \varphi \\ [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T & \text{eigenvector in the eigenvalue problem (35)} \\ [\mathbf{\hat{a}}^{(J)}, \mathbf{\hat{b}}^{(J)}]^T, [\mathbf{\hat{a}}_{k}^{(J)}, \mathbf{\hat{b}}_{k}^{(J)}]^T & \text{normalized eigenvector } [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T \\ \mathbf{c}, c_J & \text{internal length scale parameter [length]^2} \\ \mathbf{C}_{1k}, \mathbf{C}_{2k} & \text{matrices appearing in the expression for the near-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}_{44}^{'}, \mathbf{C}_{24}^{'}, & \text{matrices appearing in the expression for the derivative of the matrix } \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{\varphi}^{(J)} \\ \mathbf{M}_{\varphi}^{(J)}, \mathbf{M}_{\varphi}^{(J)} \end{bmatrix} \end{aligned} $	for the asymptotic displacements $\mathbf{u}^{(J)}$		components		
$ \begin{array}{ll} \mathbf{B}^{(J)} \text{ with respect to } \varphi \\ [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T & \text{eigenvector in the eigenvalue problem (35)} \\ [\mathbf{\hat{a}}^{(J)}, \mathbf{\hat{b}}^{(J)}]^T, [\mathbf{\hat{a}}_k^{(J)}, \mathbf{\hat{b}}_k^{(J)}]^T & \text{normalized eigenvector } [\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T \\ c, c_I & \text{internal length scale parameter [length]}^2 \\ \mathbf{C}_{1k}, \mathbf{C}_{2k} & \text{matrices appearing in the expression for the near-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{L} \\ \mathbf{L}_{2k}' & \mathbf{L}_{2k}' & \mathbf{L}_{2k}' & \mathbf{L}_{2k}' & \mathbf{L}_{2k}' \\ \mathbf{L}_{2k}' & \mathbf$	$\mathbf{A}_{,\varphi}^{(J)}, \mathbf{B}_{,\varphi}^{(J)}$ derivative of the elements of the matrices $\mathbf{A}^{(J)}$ and		$\mathbf{m}_{arphi}^{(J)*}, m_{arphi}^{(J)}$	$d^{(0)*}$ complementary dipolar stress vector \mathbf{m}_{φ} and its	
$ \begin{bmatrix} \mathbf{a}^{(J)}, \mathbf{b}^{(J)} \end{bmatrix}^{T} \text{ eigenvector in the eigenvalue problem (35)} \\ \begin{bmatrix} \mathbf{\hat{a}}^{(J)}, \mathbf{\hat{b}}^{(J)} \end{bmatrix}^{T}, \begin{bmatrix} \mathbf{\hat{a}}^{(J)}_{k}, \mathbf{\hat{b}}^{(J)}_{k} \end{bmatrix}^{T} \text{ normalized eigenvector } \begin{bmatrix} \mathbf{a}^{(J)}, \mathbf{b}^{(J)} \end{bmatrix}^{T} \\ c, c_{J} \text{ internal length scale parameter } [\text{length}]^{2} \\ \mathbf{C}_{1k}, \mathbf{C}_{2k} \text{ matrices appearing in the expression for the near-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} \text{ matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} $	$\mathbf{B}^{(J)}$ with respect to φ		compone	ents	
$ \begin{bmatrix} \hat{\mathbf{a}}^{(J)}, \hat{\mathbf{b}}^{(J)} \end{bmatrix}^{T}, \begin{bmatrix} \hat{\mathbf{a}}^{(J)}_{k}, \hat{\mathbf{b}}^{(J)}_{k} \end{bmatrix}^{T} \text{ normalized eigenvector } \begin{bmatrix} \mathbf{a}^{(J)}, \mathbf{b}^{(J)} \end{bmatrix}^{T} \\ \mathbf{c}, c_{I} & \text{internal length scale parameter } [\text{length}]^{2} \\ \mathbf{C}_{1k}, \mathbf{C}_{2k} & \text{matrices appearing in the expression for the near-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{A} \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{A} \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{A} \mathbf{u} \\ \mathbf{C}'_{1k}, \mathbf{C}'_{2k} & \text{matrices appearing in the expression for the deri-tip displacement jump across the crack } \mathbf{A} \mathbf{u} \\ \mathbf{D}'_{2k}, \mathbf{D}'_{2k} & \mathbf{U} \\ \mathbf{D}'_{2k}, \mathbf{U}'_{2k} & \mathbf{U} \\ \mathbf{U}'_{2k}, \mathbf{U}'_{2k} & \mathbf{U} \\ \mathbf{U}'_{2k}, \mathbf{U}'_{2k} & \mathbf{U}'_{2k} & \mathbf{U}'_{2k} \\ \mathbf{U}'_{2k}, \mathbf{U}'_{2k} & \mathbf{U}$	$[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$ eigenvector in the eigenvalue problem (35)		$\mathbf{M}_{r}^{(J)},\mathbf{N}_{r}^{(J)}$	matrices of the angular functions in the expression	
c, c_j internal length scale parameter [length] ² C_{1k}, C_{2k} matrices appearing in the expression for the near- tip displacement jump across the crack $\Delta \mathbf{u}$ C'_{1k}, C'_{2k} matrices appearing in the expression for the deri- C'_{1k}, C'_{2k} matrices appearing in the expression for the deri-	$[\hat{\mathbf{a}}^{(J)}, \hat{\mathbf{b}}^{(J)}]^T$, $[\hat{\mathbf{a}}^{(J)}_k, \hat{\mathbf{b}}^{(J)}_k]^T$ normalized eigenvector $[\mathbf{a}^{(J)}, \mathbf{b}^{(J)}]^T$		(1) (1)	for the asymptotic dipolar stresses \mathbf{m}_r	
$ \begin{array}{ccc} \mathbf{C}_{1k}, \mathbf{C}_{2k} & \text{matrices appearing in the expression for the near-tip displacement jump across the crack \Delta \mathbf{u} \\ \mathbf{C}_{1k}', \mathbf{C}_{2k}' & \text{matrices appearing in the expression for the deri-} \end{array} $	c, c_J	internal length scale parameter [length] ²	$\mathbf{M}_{\varphi}^{(j)}, \mathbf{N}_{\varphi}^{(j)}$, $\mathbf{M}_{y}^{(j)}$, $\mathbf{N}_{y}^{(j)}$ matrices of the angular functions in the	
tip displacement jump across the crack $\Delta \mathbf{u}$ and \mathbf{m}_y , respectively $\mathbf{C}'_{1,v}$ $\mathbf{C}'_{2,v}$ matrices appearing in the expression for the deri- $\mathbf{M}_1^{(J)} \mathbf{M}_2^{(J)}$ submatrices of the matrix $\mathbf{M}_2^{(J)}$	C_{1k}, C_{2k}	matrices appearing in the expression for the near-	ond	expression for the asymptotic dipolar stresses \mathbf{m}_{φ}	
$\mathbf{U}_{1}, \mathbf{U}_{2}$ matrices appearing in the expression for the deri- $\mathbf{M}_{1}^{*}, \mathbf{M}_{2}^{*}$ submatrices of the matrix \mathbf{M}_{2}^{*}	al -1	tip displacement jump across the crack $\Delta \mathbf{u}$	and \mathbf{m}_y ,	respectively $respectively$	
$\varphi_{1k}, \varphi_{2k}$ matrices appearing in the dipresent for the data $1, 2, 2, \dots, q$	C'_{1k}, C'_{2k}	matrices appearing in the expression for the deri-	$\mathbf{M}_{1}^{c}, \mathbf{M}_{2}^{c}$	submatrices of the matrix $\mathbf{M}_{\varphi}^{\varphi'}$	
value of the near-tip displacement jump across the $N_1^{(j)}, N_2^{(j)}$ submatrices of the matrix $N_{\varphi}^{(j)}$		vative of the near-tip displacement jump across the	$N_1^{(0)}, N_2^{(0)}$	submatrices of the matrix $\mathbf{N}_{\varphi}^{(j)}$	
$c_{rack} \sigma_y \Delta \mathbf{u}$ n outer normal unit vector	C	CTACK $\sigma_y \Delta \mathbf{u}$	n O	outer normal unit vector	
c_{14} operators of the boundary conditions on fraction- O_{14} operators of the transmission conditions at the bonded interface	C ₁₄	free crack faces	014	bonded interface	

- free crack faces $C_{1...4}^+$ operators of the boundary conditions on traction-
- free crack faces of the adjoint problem
- *E*_J Young modulus

2

 $O_{1...4}^+$

 $p,\,p_k$

operators of the transmission conditions at the

characteristic exponent of the asymptotic field

bonded interface of the adjoint problem

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