International Journal of Engineering Science 133 (2018) 99–108

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Free vibrations of elastic beams by modified nonlocal strain gradient theory

A. Apuzzo[a], R. Barretta[b,*], S.A. Faghidian[c], R. Luciano[a], F. Marotti de Sciarra[b]

[a] Department of Civil and Mechanical Engineering, University of Cassino and Southern Lazio, via G. Di Biasio 43, Cassino, FR 03043, Italy
[b] Department of Structures for Engineering and Architecture, University of Naples Federico II, via Claudio 21, Naples 80125, Italy
[c] Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

ARTICLE INFO

Article history:
Received 12 August 2018
Revised 30 August 2018
Accepted 3 September 2018

Keywords:
Bernoulli–Euler nano-beam
Axial vibrations
Flexural vibrations
Nonlocal strain gradient elasticity
Constitutive boundary conditions
NEMS

ABSTRACT

Size-dependent axial and flexural free vibrations of Bernoulli–Euler nano-beams are investigated by the modified nonlocal strain gradient elasticity model presented in (Barretta & Marotti de Sciarra, 2018). The corresponding elastodynamic problem, with the natural constitutive boundary conditions, is solved by an effective analytical methodology. Axial and flexural fundamental frequencies are determined for cantilever and fully-clamped nano-beams. Effects of nonlocal and gradient scale parameters on fundamental frequencies are examined and compared with those obtained by Eringen's local/nonlocal mixture model. New benchmarks are found for vibrations of beams. The adopted nonlocal strain gradient model, with the appropriate constitutive boundary conditions, is capable of capturing both softening and stiffening dynamical responses. Accordingly, it provides an advantageous approach for design and optimization of a wide range of nano-scaled beam-like components of Nano-Electro-Mechanical-Systems (NEMS).

1. Introduction

The recent challenges in nano-engineering have increased the necessity to rigorously examine the mechanical behavior of nano-devices. Beam-like elements are fundamental structural parts of modern nano-electro-mechanical systems (NEMS) which exhibit size effects. Therefore, a profound understanding of analytical and numerical models is required for appropriate assessment and design of NEMS (Kumar, Singh, Hui, Feo, & Fraternali, 2018; Ramsden, 2016; Zhang & Hoshino, 2014). The analysis of nano-structures has become a topic of major concern in the current literature and an extensive variety of constitutive models is exploited in the community of Engineering Science to assess size phenomena. Instances are the following. Eringen's nonlocal differential equation is employed to examine pull-in instabilities of nano-beams (Ouakad & Sedighi, 2016; Sedighi & Sheikhanzadeh, 2017; Sedighi, Keivani, & Abadyan, 2015; Sedighi, Daneshmand, & Abadyan, 2015, 2016), vibration of nano-rods (Numanoğlu, Akgöz, & Civalek, 2018), nonlinear functionally graded nano-plates (Srividhya, Raghu, Rajagopal, & Reddy, 2018) and wrinkling hierarchy in graphene (Zhao, Guo, & Lu, 2018). An enhanced Eringen's differential law was proposed by Barretta, Feo, Luciano, and Marotti de Sciarra (2016) and applied to torsion (Apuzzo, Barretta, Čanađija et al., 2017) and flexure (Barretta, Feo, Luciano, Marotti de Sciarra, and Penna, 2016; Demir & Civalek, 2017). Eringen's local/nonlocal two-phase integral methodologies are exploited to study static (Acierno, Barretta, Luciano, Marotti de Sciarra, & Russo, 2017) and dynamic axial behaviors of nano-rods (Zhu & Li, 2017a), static flexure of slender (Fernández-Sáez, Zaera, Loya, & Reddy, 2016;

* Corresponding author.
E-mail addresses: a.apuzzo@unicas.it (A. Apuzzo), rabarret@unina.it (R. Barretta), luciano@unicas.it (R. Luciano), marotti@unina.it (F. Marotti de Sciarra).

https://doi.org/10.1016/j.ijengsci.2018.09.002
0020-7225/

Koutsoumaris, Eptaimeros, & Tsamasphyros, 2017; Wang, Zhu, & Dai, 2016) and stubby nano-beams (Wang, Huang, Zhu, & Lou, 2018), as well as free vibrations (Fernández-Sáez & Zaera, 2017; Khanik, 2018) and buckling phenomena (Zhu, Wang, & Dai, 2017). Also, strain gradient models are employed for Bernoulli–Euler (Akgöz & Civalek, 2015; Barretta, Brčić, Čanađija, Luciano, & Marotti de Sciarra, 2017), Timoshenko (Marotti de Sciarra & Barretta, 2014) and shear deformable nano-beams (Akgöz & Civalek, 2014a), stability analysis of carbon nanotubes (Akgöz & Civalek, 2017a), dynamic torsion of nano-beams (Sedighi, Koochi, Keivani, & Abadyan, 2017), vibrations of three-dimensionally graded nano-beams (Hadi, ZamaniNejad, & Hosseini, 2018) and flexoelectric curved micro-beams (Qi, Huang, Fu, Zhou, & Jiang, 2018). Couple stress theories are utilized for buckling analysis (Akgöz & Civalek, 2014b; Jiao & Alavi, 2018; Taati, 2018), dynamics of elastic and viscoelastic nano-beams (Akgöz & Civalek, 2017b,2018; Attia & Abdel Rahman, 2018; Ghayesh, 2018; Ghayesh, & Farokhi, 2018) and Lamb wave dispersion in nano-plates (Ghodrati, Yaghootian, Ghanbar Zadeh, & Sedighi, 2018). Also, nonlocal strain gradient theory is used to study extension (Zhu & Li, 2017b), flexure (Li & Hu, 2016; Li, Tang, & Hu, 2018), vibration (Li, Li, & Hu, 2016; Lu, Guo, & Zhao, 2017), buckling of nano-beams (Li & Hu, 2015), dynamics of nano-tubes (Ghayesh & Farajpour, 2018; She, Ren, Yuan, & Xiao, et al., 2018) and nano-plates (Lu, Guo, & Zhao, 2018). Nonlinear analyses of Reissner nonlocal strain gradient beams are carried out in (Faghidian, 2018a) by using the variational formulation in (Faghidian, 2018b). Further overviews can be found in (Aifantis, 2016; Rafii-Tabar, Ghavanloo, & Fazelzadeh, 2016; Thai, Vo, Nguyen, & Kim, 2017). However, two basic pure nonlocal elasticity models are available in literature, depending on whether strain-driven or stress-driven formulations are adopted. The strain-driven nonlocal elasticity theory was introduced by Eringen (1983), wherein the stress field is the output of the integral convolution between elastic strain field and a smoothing kernel. Eringen's integral model (EIM) can be replaced for unbounded continua with a differential law which is commonly named Eringen differential model (EDM). EIM cannot be exploited to assess size effects in structural problems in bounded domains, due to the confliction between constitutive boundary conditions and equilibrium (Romano, Barretta, Diaco, & Marotti de Sciarra, 2017; Romano, Barretta, & Diaco, 2017; Romano, Luciano, Barretta, & Diaco, 2018). Such an ill-posedness may be partly removed by the mixture Eringen integral model (MEIM) (Eringen, 1972, 1987), as thoroughly discussed in (Romano, Barretta, & Diaco, 2017; Romano, Luciano, Barretta, & Diaco, 2018).

The stress-driven nonlocal integral model (SDM) was conceived by Romano and Barretta (2017a), where source and output fields of Eringen's strain-driven integral law are swapped. The nonlocal elastic strain field is therefore defined as output of the integral convolution between stress field and a smoothing kernel. Unlike EIM, SDM in bounded domains leads to well-posed nano-structural problems of engineering interest (Romano & Barretta, 2017b). The stress-driven nonlocal integral approach has been effectively adopted in a variety of elastostatic, elastodynamic and thermoelastic problems of nano-mechanics (Apuzzo, Barretta, Luciano, Marotti de Sciarra, & Penna, 2017; Barretta, Čanađija, Feo, et al., 2018; Barretta, Čanađija, Luciano, & Marotti de Sciarra, 2018; Barretta, Diaco, et al., 2018; Barretta, Fazelzadeh, Feo, Ghavanloo, & Luciano, 2018; Barretta, Faghidian, & Luciano 2018; Barretta, Faghidian, Luciano, Medaglia, & Penna, 2018a; Barretta, Luciano, Marotti de Sciarra, & Ruta, 2018; Mahmoudpour, Hosseini-Hashemi, & Faghidian, 2018). The stress-driven nonlocal integral law, convexly combined with the local elastic law, leads to a mixture stress-driven integral model (MSDM) which provides also a viable approach to capture size effects in bounded nano-structures (Barretta, Fabbrocino, Luciano, & Marotti de Sciarra, 2018; Barretta, Faghidian, Luciano, Medaglia, & Penna, 2018b). Also, Eringen's differential model (EDM) and strain gradient elasticity theory (SGT) were first unified by Aifantis (2003, 2011) to conceive the nonlocal strain gradient (NSG) elasticity theory. Lim, Zhang, and Reddy (2015) then combined the Eringen integral model (EIM) with the NSG to formulate a higher-order nonlocal theory. To include higher-order strain gradient effects within Eringen's integral model (EIM), a second-order integro-differential nonlocal theory was recently contributed within a thermodynamic framework by Faghidian (2018c). According to nonlocal strain gradient theory, the stress field is the sum of two integral convolutions. The former one is performed between strain field and a smoothing kernel depending on a nonlocal parameter. The latter one, involving also a gradient parameter, is the gradient of the convolution between strain gradient field and a smoothing kernel. Such an integral constitutive law, if formulated on unbounded domains and equipped with Helmholtz's bi-exponential kernels, may be correctly replaced with a differential relation, due to the tacit fulfillment of constitutive boundary conditions of vanishing at infinity. However, the nonlocal strain gradient problems of technical interest involve bounded domains, and hence, suitable constitutive boundary conditions are to be prescribed to close the constitutive law. This key issue has been solved in (Barretta & Marotti de Sciarra, 2018). It has been also pointed out that the differential law associated with the nonlocal strain gradient integral model of elasticity, equipped with appropriate constitutive boundary conditions, leads to well-posed nano-engineering problems.

In the present study, axial and flexural free vibrations of Bernoulli–Euler nano-beams are formulated by modified nonlocal strain gradient integral theory presented in (Barretta & Marotti de Sciarra, 2018), equipped with the appropriate constitutive boundary conditions. Fundamental frequencies are analytically determined for cantilever and fully-clamped beams and compared with those obtained by Eringen's local/nonlocal mixture model. The modified nonlocal strain gradient dynamical model, is demonstrated to be capable of exhibiting both softening and stiffening structural behaviors, and therefore, provides a convenient approach for assessment and design of a wide range of nano-devices exploited in NEMS applications. New benchmarks are also detected for axial and flexural free vibrations of nano-beams.