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An improved lattice hydrodynamic model accounting for the effect of "backward looking" and flow integral

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HIGHLIGHTS

- A new model is proposed considering the effect of "backward looking" and flow integral.
- The stability analysis is carried out.
- The mKdV equation is deduced to describe traffic congestion.

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ABSTRACT

In order to investigate the effect of "backward looking" and flow integral upon traffic flow, an improved lattice hydrodynamic model has been developed. The stability condition is obtained by the use of linear stability analysis. The result of stability analysis demonstrate that both the "backward looking" and flow integral play an important role in enhancing the stability of traffic flow. The mKdV equation is deduced by using the nonlinear theory, which demonstrates that traffic congestion can be described by the solution of mKdV equation. Numerical simulations are explored to study how "backward looking" and flow integral influence the stability of traffic flow. Numerical results verify that the traffic flow stability can be efficiently improved with the consideration of the above two factors.

1. Introduction

In recent years, with the increasing number of vehicles on the road, traffic congestion has attached more and more attention of scholars, the traffic problems become more and more serious. Traditional traffic approaches mainly focus on reproducing the flow-density relationship and the unstable traffic region. However, the research indicates that as for the traffic models, it is more important to describe the nonlinear phenomena and characteristics recently. Therefore, to investigate the properties of traffic congestions, a considerable variety of traffic models [1–39] have been proposed, such as car-following models [10–24], cellular automation models [25–30], macro traffic models [31–34] and lattice hydrodynamic models [35–38] in past few decades. Presently, the traffic wave is one of the most important problems.

In 1998, with the idea of car-following theory, Nagatani [39] proposed the first lattice hydrodynamic model. By using the linear stability theory and nonlinear analysis method, Nagatani found the neutral stability line and the mKdV equation successfully and derived the solution of kink density wave in the mKdV equation. Based on the model proposed by Nagatani, many extended versions have been developed with the consideration of different factors [40–43] like backward looking

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effect, driver's memory during a period of time, driver's desired average flux, and so on. Considering two-dimensional traffic flow, Nagatani extended his work to two-dimensional lattice hydrodynamic model in Ref. [44]. All the above works undoubtedly enrich the traffic flow theory and make lattice hydrodynamic models more realistic.

In actual traffic, more and more factors influence the stability of traffic flow in our life. For example, drivers always look at the following vehicle as well as the preceding vehicle, Ge et al. [45] considered this factor in traffic flow models. After that, the effect of backward looking was universally studied by considering different factors [46–50]. In order to improve the stability of Nagatani's hydrodynamic model, Tian et al. [51] proposed a new lattice model by incorporating the density difference information of two successive lattices into the original lattice hydrodynamic model. In Tian's model, the density information used for traffic state adjustment is happened at the present time. Actually, as the driving process is continuous, the regulation of traffic state is affected by the historical traffic information inevitably. In 2015, Redhu et al. [52] carried out the DFC (Delayed-feedback control) method for lattice hydrodynamic model considering the flux change in adjacent time. But, when the time interval between the present time and the historical time becomes larger, the influence of historical traffic information on traffic information is a continuous weaker process. So it is not enough to reveal the impact of historical traffic information on traffic flow, when only using the traffic information at a fixed historical time. However, the traffic evolution feature influenced by the integration of continuous historical flow information has not been studied so far. Based on this, a new lattice hydrodynamic model accounting for the effect of backward looking and flow integral will be presented to investigate its influence on traffic flow.

This paper is organized as follows. In the following section, the new lattice hydrodynamic model is presented. The linear stability analysis for the new model is discussed in Section 3. In Section 4, we deduce the mKdV equation by nonlinear analysis method and obtain the kink–antikink solution of the mKdV equation to describe the propagation behavior of traffic density waves. Numerical simulations are carried out to validate the analytic results in Section 5 and conclusions are given in Section 6.

2. The extended lattice hydrodynamic model

In 1998, Nagatani [39] proposed a lattice hydrodynamic model firstly:

$$\partial_t \rho + \partial_x \rho v = 0 \tag{1}$$

$$\partial_t \rho v = a\rho_0 V\left(\rho\left(x+\delta\right)\right) - a\rho v \tag{2}$$

where ρ_0 is the average density, a represents driver's sensitivity, ρ and v respectively denote the traffic density and velocity, δ indicates the average headway that is the inverse of ρ_0, $\rho(x+\delta)$ is the local density at position $x+\delta$, and V is the optimal velocity which is determined by the traffic density.

Then, the above model is modified with dimensionless space x (let $x^* = x/\delta$, and x^* is indicated as x hereafter) and rewritten with its lattice version as follows:

$$\partial_t \rho_j v_j = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j \tag{3}$$

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{4}$$

where j denotes the lattice number, ρ_j and v_j represent the local density and local velocity of lattice j at time t, respectively.

On the basis of lattice model proposed by Nagatani, Ge et al. [45] presented the backward looking lattice hydrodynamic (for short, BL-LH) model. The current equation is:

$$\partial_t \rho_j v_j = a\rho_0 V_F\left(\rho_{j+1}\left(t\right)\right) + H\left(\overline{\rho} - \rho_{j+1}\left(t\right)\right)\cdot H\left(\rho_{j-1}\left(t\right) - \rho_c\right) a\rho_0 V_B\left(\rho_{j-1}\left(t\right)\right) - a\rho_j v_j \tag{5}$$

where $H(\cdot)$ is the Heaviside function, $H\left(t\right) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$. $\overline{\rho}$ is a parameter which is close to 1 and ρ_c is the safety density. As the density is greater than ρ, the backward looking effect will disappear. Moreover, as we consider the density less than a certain value, which is set as the safety density ρ_c, the backward looking effect will also disappear. That is to say, only when the density is between $(\rho_c, \overline{\rho})$, the backward looking effect will play its role.

The OV function used by Nagatani is adopted as the "forward looking" one

$$V_F(\rho_{j+1}) = \tanh(\frac{2}{\rho_0} - \frac{\rho_{j+1}(t)}{\rho_0^2} - \frac{1}{\rho_c}) + \tanh(\frac{1}{\rho_c}) \tag{6}$$

and the following OV function is given as the "backward looking" one

$$V_B(\rho_{j-1}) = r\left[-\tanh(\frac{2}{\rho_0} - \frac{\rho_{j-1}(t)}{\rho_0^2} - \frac{1}{\rho_c}) + \tanh(\frac{1}{\rho_c})\right] \tag{7}$$

where ρ_o represents the initial density and r indicates a positive constant standing for the relative role of the "backward looking" OV function $V_B(\Delta\rho_{j-1})$.