Physica A 513 (2019) 465–476

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Mean-field flow difference model with consideration of on-ramp and off-ramp

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HIGHLIGHTS

- An extended lattice hydrodynamic model is proposed by incorporating mean-field flow difference.
- Applying the linear stability theory, the new model's linear stability is obtained.
- Through nonlinear analysis, the mKdV equation is derived.

ARTICLE INFO

Article history:
Received 23 May 2018
Received in revised form 2 August 2018
Available online xxxx

Keywords:
Traffic flow
On-ramp and off-ramp
Mean-field flow difference

ABSTRACT

In this paper, an extended lattice hydrodynamic model is proposed by incorporating the effects of mean-field flow difference, on-ramp and off-ramp. The linear stability condition is obtained through linear stability theory and the mKdV equation is derived by using nonlinear analysis method. Therefore, the propagation behavior of traffic jam can be described by the kink–antikink soliton solution of the mKdV equation. Furthermore, the theoretical findings are verified with the use of numerical simulation which confirms that the stability of traffic flow can be efficiently improved with the consideration of the effects of mean-field flow difference, on-ramp and off-ramp.

1. Introduction

With the vigorous growth of vehicles, traffic jam has attracted considerable attention of scholars from the point of traffic efficiency, traffic safety and energy consumption. In order to solve the traffic problem, a considerable variety of traffic models have been proposed, including car-following models [1–29], cellular automation models [30–34], macro traffic models [35–39] and lattice hydrodynamic models [40,41]. Among them, the lattice hydrodynamic model firstly proposed by Nagarani [42] in view of the idea of car-following theory. To describe traffic jam as a kink–antikink density wave without taking some factors into consideration, he derived the mKdV equation. Subsequently, a multitude of extended lattice hydrodynamic models [43–50] have been proposed by taking various kinds of real traffic information into account such as the flux difference effect, the density difference effect, driver's aggressive characteristics, and so on.

In recent years, the influence of on-ramp or off-ramp on the traffic stability of the main road have been studied based on the cellular automaton models [51–57]. Empirical observation of freeway traffic shows that the phase transition from freely moving flow to traffic congestion occurs mostly at on-ramp bottleneck and the off-ramp usually can alleviate the seriousness of traffic congestion of the main road. Furthermore, Zhang et al. [58] considered the influences of on-ramp and off-ramp in

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https://doi.org/10.1016/j.physa.2018.09.026
0378-4371/

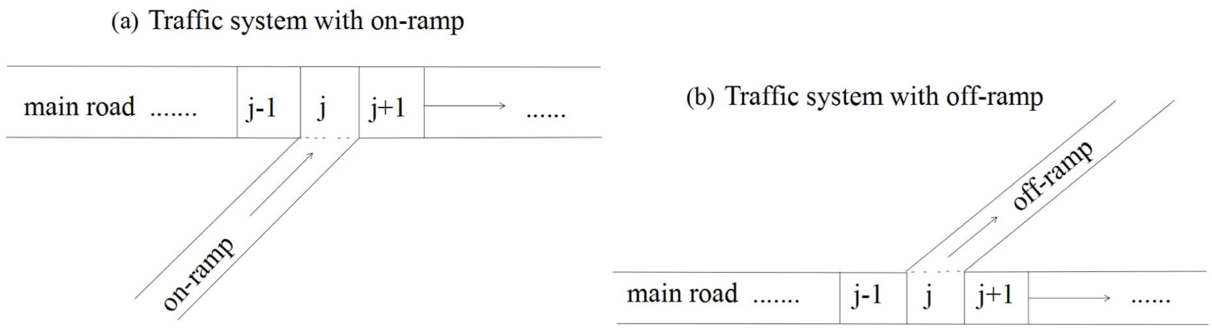

Fig. 1. The schematic of traffic system with on-ramp or off-ramp (ρ, a).

the lattice hydrodynamic model. However, the effect of mean-field flow difference was not taken into consideration in this models. In view of this, an extended lattice hydrodynamic model is proposed to investigate the effects of mean-field flow difference, on-ramp and off-ramp in this paper.

This paper is organized as follows. In Section 2, the extended model is formulated by incorporating the effects of mean-field flow difference, on-ramp and off-ramp. Therefore, the stability condition is derived with the linear theory in Section 3. In Section 4, the mKdV equation is obtained by using the method of nonlinear analysis. For the sake of demonstration of theoretical results, numerical simulation is carried out in Section 5. Conclusions are drawn in Section 6.

2. The extended lattice hydrodynamic model

Fig. 1 shows the schematic of traffic system with on-ramp or off-ramp. In Fig. 1(a), the traffic density of lattice $j-1$ of the main road is smaller than that of lattice j with the traffic flow of the on-ramp injects into the main road at lattice j. Consequently, we can define the traffic incoming flow at lattice j as $\beta\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j-1})$. Similarly, it is clear that the traffic density of lattice $j+1$ is bigger than that of lattice j with the traffic flow of the main road pouring from the off-ramp at lattice j in Fig. 1(b). The outgoing flow at lattice j can be defined as $\gamma\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j+1})$. Based on the original lattice hydrodynamic model, an extended lattice hydrodynamic model is proposed by incorporating the effects of mean-field flow difference, on-ramp and off-ramp. Therefore, the motion equation is given as follows:

$$\partial_t(\rho_j v_j)=a\rho_o V(\rho_{j+1})-a\rho_j v_j+ak(\frac{1}{n}\sum_{l=0}^{n-1}\rho_{j+l}v_{j+l}-\rho_j v_j), \tag{1}$$

$$\partial_t\rho_j+\rho_o(\rho_j v_j-\rho_{j-1}v_{j-1})=\beta\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j-1})+\gamma\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j+1}), \tag{2}$$

where $\frac{1}{n}\sum_{l=0}^{n-1}\rho_{j+l}v_{j+l}$ depicts an average effect of all lattice interactions on the whole road including the contribution of *jth* lattice. The mean-interaction term in this model like the mean-field feedback control is easily realized by modern means of information gathering in intelligent transportation system. k denotes the strength of the average effect of multi-lattice interaction or feedback gain of the mean-field feedback control. β denotes the on-ramp rate of lattice j and γ represents the off-ramp rate of lattice j. $a=\frac{1}{\tau}$ is the sensitivity which corresponds to the inverse of the delay time. $V(\rho)$ is the optimal velocity function that is assumed to be:

$$V(\rho)=\frac{v_{\max}}{2}[\tan h(\frac{2}{\rho_o}-\frac{\rho}{\rho_o^2}-\frac{1}{\rho_c})+\tan h(\frac{1}{\rho_c})], \tag{3}$$

where ρ_o is the initial density, ρ_c represents the safety density and $v_{max}=2$ is the maximal velocity. Eliminating speed v in Eqs. (1) and (2), we obtain the following density equation:

$$\begin{aligned}
&\partial_t^2\rho_j-\partial_t[\beta\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j-1})+\gamma\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j+1})]\\
&+a\rho_0^2[V(\rho_{j+1}(t))-V(\rho_j(t))]+a(k+1)\partial_t\rho_j-ak\frac{1}{n}\sum_{l=0}^{n-1}\partial_t\rho_{j+l}\\
&-a[\beta\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j-1})+\gamma\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j+1})]\\
&+ak\frac{1}{n}\sum_{l=0}^{n-1}[\beta\left|\rho_o^2V'(\rho_o)\right|(\rho_{j+l}-\rho_{j+l-1})+\gamma\left|\rho_o^2V'(\rho_o)\right|(\rho_{j+l}-\rho_{j+l+1})]\\
&-ak[\beta\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j-1})+\gamma\left|\rho_o^2V'(\rho_o)\right|(\rho_j-\rho_{j+1})]=0.
\end{aligned} \tag{4}$$