Physica A 513 (2019) 510–517

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

An extended lattice hydrodynamic model considering the delayed feedback control on a curved road

Rongjun Cheng [*], Yunong Wang

Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China
Jiangsu Key Laboratory of Urban ITS Southeast University, Nanjing 210096, China
National Traffic Management Engineering and Technology Research Centre Ningbo University Sub-centre, Ningbo 315211, China

HIGHLIGHTS

- An extended lattice hydrodynamic model is proposed considering the effect of delayed feedback control on a curved road.
- The stability of this new model is obtained by the control method.
- Numerical simulations are carried out to explore the influence of delayed feedback control on a curved road.

ARTICLE INFO

Article history:
Received 6 June 2018
Received in revised form 3 August 2018
Available online xxxx

Keywords:
traffic flow
Lattice hydrodynamic model
Delayed feedback control
Curved roads

ABSTRACT

In this paper, we investigate the effect of delayed feedback control on a curved road analytically and numerically. An extended lattice hydrodynamic model is derived on a single-lane road which includes more comprehensive information. The control method is used to obtain the stability of the new model and the critical condition for the linear steady traffic flow is deduced. Moreover the numerical simulations are carried out to verify the advantage of the proposed model with and without the delayed feedback control on a curved road and a straight road. The results are consistent with the theoretical analysis correspondingly.

1. Introduction

The fast development of transportation brings a great convenience to our daily life, more people are tend to drive the private cars to go outside. But meanwhile, with the rapid increase of automobiles, the urban traffic problems like the traffic congestion, traffic accident, air pollution and global warming, are also raised as a heated issue. Recent decades, more and more engineers and scholars are committed to solving the urban traffic problems. Up to now, various traffic flow models have been developed to disclose the nature of traffic jams from different points of views such as the car-following models [1–20], the cellular automaton models [21–23], the hydrodynamic models [24–50], and the gas kinetic models [51,52].

As one of the microscopic models, the car-following models have been widely used to analyse the characteristics of traffic flow under various conditions when Pipes [1] proposed a classical model in 1953. But the effect of driver's sensitivity has not been fully considered in the model. So in 1995, Bando et al. [6] put forward an improved car-following model called optimal velocity model (OVM, for short). Based on OVM, a multitude of car-following models have been extended with delayed feedback control from the view of control theory [7–12]. In 2006, Zhao and Gao [13] proposed a simple

---

[*] Corresponding author at: Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China.
E-mail address: chengrongjun@nbu.edu.cn (R. Cheng).

https://doi.org/10.1016/j.physa.2018.09.014

coupled-map(CM) car-following model considering the velocity feedback control to suppress the traffic jam. Subsequently, Han [14] and Ge [15] took into account the ITS with CM car-following model in 2007 and 2011 respectively. Then in 2014, Li [16] presented a dynamic collaboration model with feedback signals to suppress the traffic congestion. But up to present, not like the thorough mechanism research in the field of microscopic, the studies for macroscopic models with control theory are still infrequent.

In 1998, famous scholar Nagatani [27] proposed the lattice hydrodynamic model which firstly summarized the variational relationships between collective variables, and the traffic flow was divided as the lattice site lined one by one. Motivated by the Nagatani's research, more and more new lattice hydrodynamic models have been put forward with the consideration of different factors from the macroscopic viewpoint [29–42], but the studies touching upon the control signal in lattice hydrodynamic models are still rare. In 2015, Redhu [44] carried out the DFC method for lattice hydrodynamic model considering the flux change in adjacent time and Ge [43] presented a simple control method with the lattice hydrodynamic model by applying a decentralized delayed-feedback control. Recently, Li [45] studied the lattice hydrodynamic model performance based on delayed feedback control with a view of density change rate difference.

As is well known, not all roads are even or straight in the actual traffic, the traffic jams always happens on the curved roads and even the traffic accidents will occur. The consideration of curved roads can make the proposed model closer to real traffic condition. Up to now, some scholars have already investigated the characteristics of traffic flow on curved roads [53,54], but no studies have ever tried to analyse the traffic jams for macroscopic model considering the delayed-feedback control on curved roads which has a significant impact on traffic movement. In this paper we will propose an analytical traffic flow model based on Nagatani's model. The effects of the delayed feedback control on curved roads will be discussed analytically and numerically to investigate its influence on traffic flow.

The outlines of this paper are as follows. In Section 2, the extended lattice hydrodynamic model considering the curved roads is presented, and its stability condition is analysed. A control signal will be added into the model proposed in Section 2 and delayed feedback control theory is used to analyse the stability conditions in Section 3. In Section 4, numerical simulation is carried out for lattice hydrodynamic model with and without using delayed-feedback control signal on a curved road or a straight road. Finally, conclusion is given in Section 5.

2. Lattice hydrodynamic model on the curved roads

To study the complex mechanism behind the traffic flow, Nagatani [27] put forward the lattice hydrodynamic model which analysed the density wave of traffic flow on a unidirectional road as follows:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v \end{cases} \tag{1}$$

where ρ and $\rho(x+\delta)$ are the local density at the position of x and $x+\delta$ at time t, respectively. ρ_0 means the local average density and δ is the average space headway ($\rho_0 = \frac{1}{\delta}$). a is the sensitivity of the driver. v is the local average speed and $V(\rho)$ represents the optimal speed of traffic flow at density of ρ.

In order to make it easier to proceed the further study, Nagatani modified the equation in a discretization way as follows:

$$\begin{cases} \partial_t \rho_j + \rho_0\left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j \end{cases} \tag{2}$$

where the road is divided into N lattice sites and j indicates the site of the road on the one-dimensional lattice, ρ_j and v_j severally indicates the local density and the local average speed on site j at time t.

In the real traffic, the curved roads always tend to be the main cause of the frequent traffic accidents, especially when the speed is too fast, the risk factor will increase highly. Based on this, we integrate the curved roads factor into the lattice hydrodynamic model in which flow evolution equation is modified as

$$\begin{cases} \partial_t \rho_j + \rho_0\left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t u_j + \lambda\rho_0\left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t(\rho_j v_j) = a[m\rho_0 V(\rho_{j+1}) + n\rho_0 V(u_{j+1}) - \rho_j v_j] \end{cases} \tag{3}$$

where q represents the product of ρ and v. ρ_j is the local density on site j at time t on straight road, u_j is the local density on site j at time t on curved road, λ is the reaction coefficient which represents the driver's reaction when goes through the curved roads with different radius, m and n is the ratio of straight roads and curved roads in the traffic system, $m+n=1$. And because Eq. (2) shows the similarity with Bando's car-following model, we adopt the analogous optimal speed equation