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Phase transitions and universality in the Sznajd model with anticonformity

Matheus Calvelli [a,b], Nuno Crokidakis [b,*], Thadeu J.P. Penna [a]

[a] ICEx, Universidade Federal Fluminense, Volta Redonda/RJ, Brazil
[b] Instituto de Física, Universidade Federal Fluminense, Niterói/RJ, Brazil

HIGHLIGHTS

- Impact of anticonformity in the Sznajd model in regular lattices.
- Occurrence of order–disorder transitions.
- Universality class of the Ising model.

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ABSTRACT

In this work we study the dynamics of opinion formation in the Sznajd model with anticonformity on regular lattices in two and three dimensions. The anticonformity behavior is similar to the introduction of Galam's contrarians in the population. The model was previously studied in fully-connected networks, and it was found an order–disorder transition with the order parameter exponent $\beta = 1/2$ calculated analytically. However, the other phase transition exponents were not estimated, and no discussion about the possible universality of the phase transition was done. Our target in this work is to estimate numerically the other exponents γ and ν for the fully-connected case, as well as the three exponents for the model defined in square and cubic lattices. Our results suggest that the model belongs to the Ising model universality class in the respective dimensions.

1. Introduction

Models of opinion formation have been studied by physicists since the 80's and are now part of the new branch of physics called sociophysics. This recent research area uses tools and concepts of statistical physics to describe some aspects of social and political behavior [1–3]. From theoretical point of view, opinion models are interesting to physicists because they present order–disorder transitions, scaling and universality, among other typical features of physical systems, which called the attention of many groups throughout the world [4–15].

The standard approach to many models of opinion dynamics is to consider that the individuals are susceptibles, i.e., they can change opinion due to interactions with other individuals in the population [1,2]. In such case, it is usual to define a network of contacts among individuals and the rules of microscopic interaction. The most famous models considering opinions as discrete or continuous variables are the Sznajd model [16], the Galam model [17,18], the CODA model [8], the kinetic exchange opinion models [7,19], the Deffuant model [20], the Hegselmann–Krause model [21], among others. Susceptible agents are also known as conformist individuals. On the other hand, there are also individuals that act in an

* Corresponding author.
E-mail addresses: matheuscalvelli@id.uff.br (M. Calvelli), nuno@if.uff.br (N. Crokidakis), thadeupenna@id.uff.br (T.J.P. Penna).

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opposite way in comparison with the conformist, called anticonformists. Anticonformists are similar to conformists, since both take cognizance of the group norm. Whereas conformists agree with the norm, anticonformers disagree. Galam created the term "contrarians" to refer to those individuals [22]. The presence of such contrarians/anticonformists usually lead the model in consideration to undergo order–disorder phase transitions [1–3,23,24].

In this work we study the effects of anticonformity in the dynamics of the Sznajd model. The model is defined on square and cubic lattices, and also in the fully-connected network. Our motivation is to explore in details the critical behavior of the model, since the original work [23] only considered the mean-field formulation of the model, and it did not explored the critical behavior of the model. We estimate the critical exponents numerically for the mentioned lattices, and we verified that the model belongs to the universality class of the Ising model on the same lattices.

This work is organized as follows. In Section 2 we present the microscopic rules that define the Sznajd model with anticonformity, and our numerical results on regular lattices. Finally, our conclusions are presented in Section 3.

2. Model

The Sznajd model with anticonformity was studied recently. The authors defined it in the fully-connected graph [23]. Let us consider a set of N individuals (or agents), each one described by an Ising-like variable $o_i = +1\,(\uparrow)$ or $o_i = -1\,(\downarrow)$ ($i = 1, 2, \ldots, N$), denoting two opposite opinions, for example the vote for two distinct candidates A or B. The initial concentration of each opinion is 0.5 (disordered state). At each time step two neighbor agents (for example i and j) are chosen, and they influence a third neighbor (for example k) in the following way:

- $\uparrow\uparrow\Downarrow\rightarrow\uparrow\uparrow\Uparrow$, conformity with probability p_1
- $\downarrow\downarrow\Uparrow\rightarrow\downarrow\downarrow\Downarrow$, conformity with probability p_1
- $\uparrow\uparrow\Uparrow\rightarrow\uparrow\uparrow\Downarrow$, anticonformity with probability p_2
- $\downarrow\downarrow\Downarrow\rightarrow\downarrow\downarrow\Uparrow$, anticonformity with probability p_2

First two processes correspond to conformity, and the next two describe anticonformity. The authors in [23] defined the ratio $r = p_2/p_1$ as the relevant parameter to control the phase transition of the model. Furthermore, the authors decided to investigate the case in which $p_1 = 1$ and $p_2 \in [0, 1]$ is the only parameter of the model. In this case, the control parameter is indeed p_2, since $r = p_2$ for $p_1 = 1$. As discussed in the original work [23], it can be easily shown that contrarian behavior introduced by Galam [22] requires that $p_1 + p_2 = 1$ and therefore is a special case of anticonformity.

In order to analyze the phase transition, we usually define the public opinion as the magnetization per site of the system,

$$m = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} o_i \right| \right\rangle , \tag{1}$$

where $\langle \ldots \rangle$ denotes a disorder or configurational average, taken after a large enough number of whole-lattice sweeps.

Our objective in this work is to analyze the critical behavior of the system. In addition to the above-mentioned order parameter m, Eq. (1), we will also consider the susceptibility χ and the Binder cumulant U, that were not considered in the previous work [23]. Those quantities are defined respectively as

$$\chi = N\left(\langle m^2\rangle - \langle m\rangle^2\right) , \tag{2}$$

$$U = 1 - \frac{\langle m^4\rangle}{3\,\langle m^2\rangle^2} . \tag{3}$$

In order to estimate the critical exponents of the phase transition, we performed numerical simulations of the model for distinct population sizes N, and we considered a finite-size scaling (FSS) analysis of the numerical data, based on the standard FSS relations [25]

$$m(N) \sim N^{-\beta/\nu} \tag{4}$$

$$\chi(N) \sim N^{\gamma/\nu} \tag{5}$$

$$U(N) \sim \text{constant} \tag{6}$$

$$p_{2,c}(N) - p_{2,c} \sim N^{-1/\nu} , \tag{7}$$

that are valid in the vicinity of the transition. Based on Eqs. (4)–(7), we can estimate the critical exponents β, γ and ν. In this case we will consider separately in the following subsections three distinct cases, namely the model defined on: (i) the fully-connected graph, (ii) the two-dimensional square lattice, and (iii) the three-dimensional cubic lattice.

2.1. Fully-connected graph

In this case, we consider that each agent can interact with all others, i.e., all the three individuals (i, j, k) are randomly chosen in the population. The analytical calculations performed in [23] for the steady states of the model predicted that there