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Fuzzy weighted recurrence networks of time series

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HIGHLIGHTS

- Applications of network theory are wide-ranging.
- Weighted networks are ubiquitous in many real-life complex systems.
- New concept of fuzzy weighted recurrence networks is presented.
- Fuzzy weighted recurrence networks are more robust than unweighted recurrence networks.

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ABSTRACT

The concept of networks in the context of graph theory delineates a wide variety of real-life complex systems. The theory of networks finds its applications very useful in many scientific and intellectual domains. Weighted networks can characterize complex statistical graph properties, particularly where node connections are heterogeneous. A framework of fuzzy weighted recurrence networks of time series is presented in this letter. Popular graph measures including the average clustering coefficient and characteristic path length of fuzzy weighted recurrence networks are shown to be more robust than those of unweighted recurrence networks derived from binary recurrence plots.

1. Introduction

Applications of complex networks are pervasive in many disciplines, including natural science, computer science, engineering, life science, medicine, health, sociology, economics, and finance. Stemming from the concept of recurrence plots [1,2], research into recurrence networks of time series has opened a new direction for exploring and gaining insight into the behavior of complex systems [3,4]. Recurrence networks constructed from recurrence plots are unweighted networks. However, the existence of weighted networks is widespread in many natural relationships [5–7], where weights represented in real-life networks are heterogeneous. Thus, the use of network weights is useful for recognizing links of varying importance and influence in complex systems [7]. Yet relatively little effort has been spent on the development of methods for weighted recurrence networks. It appears that there is only one method for constructing multivariate weighted recurrence networks [8], in which the edge weights are obtained using cross recurrence plots of multiple dynamical systems; but none for univariate weighted recurrence networks.

Based on the concept of fuzzy recurrence plots [9], the formulation of univariate fuzzy weighted recurrence networks of time series is introduced herein. It is pointed out herein that the univariate scalable recurrence networks reported in [10] are also derived from fuzzy recurrence plots, but these networks are unweighted and therefore a different development with respect to the work addressed herein. The rest of this letter is organized as follows. Section 2 briefly reviews the technique for constructing an unweighted recurrence network. Section 3 presents the formulation of a fuzzy weighted recurrence network.

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Two popular graph measures known as the clustering coefficient and characteristic path length for unweighted and weighted networks are presented in Section 4. Results obtained from unweighted recurrence networks and fuzzy weighted recurrence networks are presented in Section 5. Comparisons and discussion of the graph measures of complex networks obtained from unweighted recurrence networks and fuzzy weighted recurrence networks are addressed in Section 6. Finally, Section 7 is the conclusion of the research findings.

2. Unweighted recurrence networks

An unweighted recurrence network (RN) represented by its adjacency matrix $\mathbf{A}$ is defined as [11]

$$\mathbf{A} = \mathbf{R} - \mathbf{I}, \tag{1}$$

where $\mathbf{R}$ and $\mathbf{I}$ are the $N \times N$ recurrence matrix of the recurrence plot and $N \times N$ identity matrix, respectively.

A recurrence matrix is constructed by considering the recurrences of the phase-space states $\mathbf{X} = \{\mathbf{x}\}$. In other words, a recurrence matrix is a visualization of the number of times the phase space trajectory of the dynamical system visits the same location in the phase space it has visited before. Hence, a recurrence plot, denoted as $RP = [R_{ij}]$ is defined as [2]

$$R_{ij} = \Theta(\phi - \|\mathbf{x}_i - \mathbf{x}_j\|), \; i, j = 1, \ldots, N, \tag{2}$$

where ϕ is the recurrence threshold, and Θ is the Heaviside step function, that is $\Theta = 1$ if $\|\mathbf{x}_i - \mathbf{x}_j\| \leq \phi$, or $\Theta = 0$ if $\|\mathbf{x}_i - \mathbf{x}_j\| > \phi$.

3. Fuzzy weighted recurrence networks

Let $\mathbf{X} = \{\mathbf{x}\}$ be the set of phase-space states, N a given number of clusters of the states, and a set of N fuzzy clusters, $\mathbf{V} = \{\mathbf{v}_i : i = 1, \ldots, N\}$. Fuzzy clusters can be defined as groups that contain data points, where each data point has a degree of fuzzy membership of belonging to each group (the reader is referred to [12] for detailed explanations about the concept and technical formulation of fuzzy clustering). By analogy with the inference for constructing a fuzzy recurrence plot and scalable network [9,10], a fuzzy relation $\tilde{\mathbf{R}}$ between $\mathbf{v}_i$ and $\mathbf{v}_j$, $i, j = 1, \ldots, N$, is characterized by a fuzzy membership function $\mu \in [0, 1]$, which expresses the degree of similarity of each pair $(\mathbf{v}_i, \mathbf{v}_j)$ in $\tilde{\mathbf{R}}$, and has the following three properties [13]:

1. Reflexivity: $\mu(\mathbf{v}_i, \mathbf{v}_i) = 1, \; \forall \mathbf{v}_i \in \mathbf{V}$.
2. Symmetry: $\mu(\mathbf{v}_i, \mathbf{x}) = \mu(\mathbf{x}, \mathbf{v}_i), \; \forall \mathbf{x} \in \mathbf{X}, \forall \mathbf{v}_i \in \mathbf{V}$.
3. Transitivity: $\mu(\mathbf{v}_i, \mathbf{v}_j) = \vee_{\mathbf{x}}[\mu(\mathbf{v}_i, \mathbf{x}) \wedge \mu(\mathbf{v}_j, \mathbf{x})], \; \forall \mathbf{x} \in \mathbf{X}, \forall \mathbf{v}_i, \mathbf{v}_j \in \mathbf{V}$, where the symbols $\vee$ and $\wedge$ stand for max and min, respectively.

An $N \times N$ fuzzy weighted recurrence network (FWRN) can be constructed with an associated fuzzy weighted adjacency matrix as

$$\mathbf{W} = \tilde{\mathbf{R}} - \mathbf{I}, \tag{3}$$

where $\mathbf{W}$ is an $N \times N$ adjacency matrix of edge weights, and $\mathbf{I}$ is the $N \times N$ identity matrix.

The set of N fuzzy clusters, $\mathbf{V}$, can be obtained using the fuzzy c-means algorithm (FCM) [12] as follows. Let μ_{ij} denote a fuzzy membership grade of $\mathbf{x}_i$, $i = 1, \ldots, M$, which belongs to a cluster j, $j = 1, \ldots, c$, whose center is $\mathbf{v}_j$. This fuzzy membership is calculated by the FCM as

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{c} \left[\frac{d(\mathbf{x}_i, \mathbf{v}_k)}{d(\mathbf{x}_i, \mathbf{v}_j)} \right]^{2/(m-1)}}, \tag{4}$$

where $1 \leq m < \infty$ is the weighting exponent, and $d(\mathbf{x}_i, \mathbf{v}_j)$ is used as a Euclidean distance between $\mathbf{x}_i$ and $\mathbf{v}_j$.

Using the fuzzy membership grades, each cluster center $\mathbf{v}_j$ is computed as

$$\mathbf{v}_j = \frac{\sum_{i=1}^{M} (\mu_{ij})^m \, \mathbf{x}_i}{\sum_{i=1}^{M} (\mu_{ij})^m}, \; \forall j. \tag{5}$$

The iterative procedure of the FCM is outlined as follows.

1. Given c, m, step t, $t = 0, \ldots, T$, initialize matrix $\mathbf{U}^{(t=0)} = [\mu_{ij}]^{(t=0)}$
2. Compute $\mathbf{v}_j^{(t)}, j = 1, \ldots, c$, using Eq. (5).
3. Update $\mathbf{U}^{(t+1)}$ using Eq. (4).
4. If $\|\mathbf{U}^{(t+1)} - \mathbf{U}^{(t)}\| < \epsilon$ or $t = T$, stop. Otherwise, set $\mathbf{U}^{(t)} = \mathbf{U}^{(t+1)}$ and return to step 2.