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Graphical solution for system optimum dynamic traffic assignment with day-based incentive routing strategies

Chuan-Lin Zhao[a,b,*], Ludovic Leclercq[a]

[a] Univ. Lyon, ENTPE, IFSTTAR, LICIT, Lyon F-69518, France
[b] School of Civil and Transportation Engineering, Beijing University of Civil Engineering and Architecture, Beijing 100044, China

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ABSTRACT

This paper analyzes the dynamic traffic assignment problem on a three-alternative network with day-based incentive routing strategies by using graphical solution method. It is assumed that the cumulative count curve of vehicles is known and that the arrival rate is unimodal. The dynamic system optimum (DSO) allocation lines are first drawn based on calculus of variations. Three possible optimal allocation lines are analyzed. A day-based incentive routing strategy is designed and conditions that when and how to implement the incentive scheme to realize DSO are then derived. Extension to general parallel networks is also given. Examples are presented to demonstrate the effectiveness of the scheme.

1. Introduction

Until now, many methods have been proposed to reduce the traffic congestion in both static and dynamic cases, including (1) economic policies, e.g., congestion pricing (Pigou, 1920; Evans, 1992; Yang and Huang, 2005; Laval et al., 2015; Daganzo and Lehe, 2015); tradable credits (Yang and Wang, 2011; Akamatsu and Wada, 2017; Lahlou and Wynter, 2017); rewards (Rouwendal et al., 2012); transit subsidy (Parry and Small, 2009); parking pricing (D'Acierno et al., 2006); and (2) engineering control schemes, e.g., speed limits (Knoop et al., 2010; Yang et al., 2012; Chen and Ahn, 2015); ramp metering (Papageorgiou and Kotsialos, 2002); license plate rationing (Nie, 2017); lane control (Dahlgren, 2002; Daganzo and Cassidy, 2008; Fosgerau, 2011) and so on. Although the popular congestion pricing schemes have been implemented in some cities, there are still a long way toward eliminating the public's reluctance to accept tolls. For tradable credits, to our best knowledge, it has not been implemented in any cities yet. For engineering control schemes, most studies investigate the effects of these schemes on network performance based on simulation approach. Other combined schemes (e.g., Daganzo, 1995; Daganzo et al., 2002; Basso and Jara-Díáaz, 2012; Wang et al., 2015) are also studied in literature.

For this paper we focus on the day-based incentive routing strategies that should be associated with multiple days, which is a very challenging research topic. Take two routes for example. One route is the main route *M* with lower free flow travel time. Another route is side route *S* with higher free flow travel time. Users who want to choose *M* for one-day use need to pay some credits or points which can be freely obtained by choosing *S* for a few days. Note that the concept of credit in this paper is different with the one in Yang and Wang (2011) in the following two aspects: (1) in Yang and Wang (2011) the credits that are freely obtained from the government are tradable. Users need to pay some money to obtain additional

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[*] Corresponding author at: Beijing University of Civil Engineering and Architecture, No 15 Yongyuan Road, Huangcun Town, Daxing District, Beijing, 102616, China.

E-mail addresses: zhaochuanlin@bucea.edu.cn (C.-L. Zhao), ludovic.leclercq@entpe.fr (L. Leclercq).

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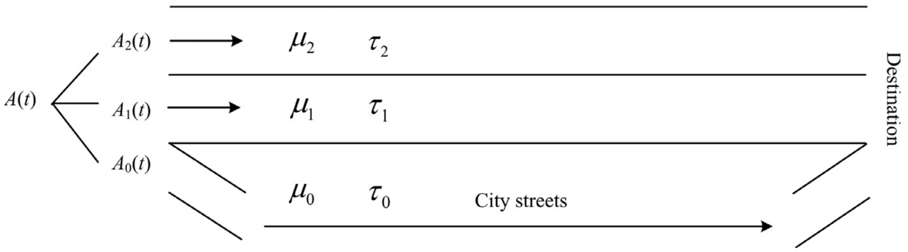

Fig. 1. Schematic of the three route network.

credits or can earn some money by selling some credits. That is, the value of the credits as one part of total trip cost will affect the users' route choice decisions; (2) while in this paper the credits or points that represent one kind of rights are freely obtained by choosing side route. With the credits or points users have the right to choose the main route. That is, the only role of the credits or points is to encourage or motivate users to choose the side route. Note that the credits are not tradable in this paper. Thus the values of the credits are not considered as one part of the route trip cost. For each user, they have to make the decisions, i.e., how many days in one period (e.g., P days) I can choose route M and how many days in one period I need to choose route S? The question of interest is how to design this day-based incentive routing strategy to realize the state of system optimum, which will be investigated in this study. That is, the whole idea is to obtain a daily flow pattern, which is close to SO while users keep follow a UE discipline. The switch is made considering day-based incentive, i.e., by providing credits for not using the optimal individual path for some days.

Another related literature to this study is about system optimum dynamic traffic assignment(SODTA) problem. A vast body of literature has been developed in this area over the past four decades (e.g., Merchant and Nemhauser, 1978a; Merchant and Nemhauser, 1978b; Friesz et al., 1989; Ghali and Smith, 1995; Ziliaskopoulos, 2000; Nie, 2011; Abdul Aziz and Ukkusuri, 2012; Carey and Watling, 2012; Zhu and Ukkusuri, 2013; Ma et al., 2014; Doan and Ukkusuri, 2015; Lu et al., 2016). Two kinds of models, i.e., discrete-time models and continuous-time models, are mostly used in the existing literature. Most discrete-time models share a similar mathematical programming structure with heuristic approaches to find the DSO solutions; while the continuous-time models use continuous optimal control method.

A persistent issue is the need to trade-off mathematical tractability with traffic realism. This paper will focus on another analytical method, i.e., graphical solution method, to find the exact DSO solutions. Note that this method belongs to continuous-time models. There are only a few publications that have approached the DSO problem by using this method. To our best knowledge, Muñoz and Laval (2006) are the first to introduce the method based on dynamic optimality conditions and calculus of variations to draw the dynamic system optimum (DSO) allocation lines. Recently, Laval et al. (2015) analyzed the effects of system optimum tolls on dynamic traffic assignment problem in a two-alternative network. As mentioned in the above two papers, the time intervals when the alternatives are used at capacity can be defined uniquely, but the allocation of the queues is not. Laval (2009) also studied the dynamic user optimum (DUO) traffic assignment problem in a simple parallel network and derived some analytical results. One important assumption is used in the above three papers that the cumulative count curve of vehicles is known and exogenous. That is, no departure time choice is considered in these papers. By relaxing the assumption, Arnott et al. (1990a) and Shen and Zhang (2009) studied both departure time and route choice problem in a parallel network.

In all, in this study we will investigate the DSO traffic assignment problem in a three-alternative network with the assumption that the cumulative arrival curve of vehicles is known based on Muñoz and Laval (2006). The logic framework of this paper includes three parts, i.e., *Step a*: Solve the DSO solutions by using graphical solution method; *Step b*: Design the routing strategies based on the DSO solutions and prove the DSO is DUE with the designed strategies; *Step c*: Extension to general parallel networks.

This rest of the paper is organized as follows: Section 2 presents the modeling assumptions and problem formulation in a three-alternative network. Section 3 illustrates the extended graphical solution method to draw the DSO allocation lines. Section 4 describes the day-based incentive routing strategies and examines the DUO equilibrium state obtained as a result of its application. Extension to general parallel networks is also given in this section, and finally Section 5 concludes the paper.

2. Problem formulation

The network consists of one origin, one destination, and three alternatives (i.e., a freeway with two lanes and a city-street); see Fig. 1. Note that the network, i.e., one-lane freeway with city-street alternative, is often used to analyze classical morning commute problem, but here we add the opportunity to consider the lane-allocation strategy on the freeways, especially to analyze the effects of day-based incentive routing strategies. Suppose that we know the cumulative count curve $A(t)$ of vehicles entering a freeway segment with two lanes or city-street with longer free flow travel time. Let the corresponding flow be $\lambda(t) = \dot{A}(t)$. As shown in Fig. 1, the cumulative count curve of vehicles at time t using route r is denoted $A_r(t)$ and the flow, $\lambda_r(t) = \dot{A}_r(t)$. Clearly, we have $\lambda(t) = \lambda_2(t) + \lambda_1(t) + \lambda_0(t)$. Another assumption is used throughout the paper that