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# Networks of fixed-cycle intersections

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A B S T R A C T

We present an algorithmic method for analyzing networks of intersections with static signaling, with as primary example a line network that allows traffic flow over several intersections in one main direction. The method decomposes the network into separate intersections and treats each intersection in isolation using an extension of the fixed-cycle traffic-light (FCTL) queue. The network effects are modeled by matching the output process of one intersection with the input process of the next (downstream) intersection. This network analysis provides insight into wave phenomena due to vehicles experiencing progressive cascades of green lights and sheds light on platoon forming in case of imperfections. Our algorithm is shown to match results from extensive discrete-event simulations and can also be applied to more complex network structures.



## 1. Introduction

Intersections are natural bottlenecks and crucially influence the dynamics of urban traffic. While intersections can be studied in isolation (Darroch, 1964; Heidemann, 1994; Mung et al., 1996; Tachet et al., 2016; van Leeuwaarden, 2006), the larger picture of networks of multiple intersections is increasingly important, also in view of the rapid growth of urbanization (Bettencourt, 2013; Çolak et al., 2016). This paper contributes to the theoretical underpinning of traffic networks by extending classical models for isolated intersections to models for networks of intersections with static signaling.

Think of a series of traffic lights designed to let traffic flow over several intersections in one main direction. Any vehicle traveling along (at an approximate prescribed speed) wants to meet a progressive cascade of green lights, and not have to stop at intersections. In practical use, only a group of vehicles – referred to as *platoon* – can pass the intersection before the time band is interrupted to give way to other traffic flows. The platoon sizes are governed by the signal times. Our method to model such situations consists of two ingredients: An extension of a classical queueing model for one isolated intersection that can deal with correlated input and that allows for a detailed characterization of the output process of an intersection, and an algorithm for network analysis that decomposes the series of queues into multiple isolated queues. While interesting

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in their own right, and likely to find more applications in transportation science, network analysis of intersections requires the delicate combination of both ingredients. We now discuss each of them separately.

**Queueing model for one intersection.** The classical model for an isolated intersection that we adopt and extend in this paper is the fixed-cycle traffic-light (FCTL) queue; one of the most well-studied stochastic models in traffic engineering (Darroch, 1964; McNeill, 1968; Newell, 1965; Webster, 1958). Vehicles arrive to an intersection controlled by a traffic light and form a queue. The time scale is divided into time intervals of unit length, and the traffic light alternates between red and green periods of fixed durations $r$ and $g$ time units. Delayed vehicles depart during the green period, where it takes one time unit for each delayed vehicle to depart; departures thus occur at equally spaced times until either the queue dissipates or the green phase terminates. Darroch (1964) obtained the probability generating function (pgf) of the steady-state overflow queue (the number of vehicles waiting in front of the traffic light at the end of a green period) and the pgf of the steady-state delay was obtained in van Leeuwaarden (2006). Hence, all information about the distribution of the steady-state overflow queue and steady-state delay in the FCTL queue can be obtained from the results in Darroch (1964) and van Leeuwaarden (2006), including all moments of the steady-state queue length and delay, and the distribution of the output process (the way vehicles leave the intersection). The output process of the first intersection is of crucial importance for the present paper, because it will serve as input process for some other signalized intersection. Moreover, the output process of a second intersection serves as input for a third intersection, and so forth. This network effect acts as a filter that modifies, and perhaps streamlines, the arrival process at consecutive intersections. Therefore, we shall address in this paper the technical challenge of extending the classical FCTL queue to allow for nonuniform and hence time-dependent correlated arrival processes. We call this extended model the generalized FCTL queue.

**Network algorithm.** A network of intersections with correlated input and output processes poses completely new challenges. Several approaches have been developed so far, but they are not based on the on the FCTL queue. Existing techniques to analyze isolated intersections are known to be inadequate in network settings, due to the highly correlated arrival process (Dion et al., 2004; Heidemann, 1994). Even techniques that are not based on a steady-state analysis, such as the probabilistic approach by Viti and Van Zuylen (2010) that does not require arrivals in different time slots to be identically distributed, still assume independent arrivals. We therefore develop an approximation scheme to evaluate the system performance based on decomposition. While this approach has been successfully applied to classic queueing networks (Bitran and Dasu, 1992; Kühn, 1979; Whitt, 1983), a network of generalized FCTL queues poses additional challenges due to the non-synchronized cyclic structures and inherently correlated arrival processes. We decompose the network into isolated generalized FCTL queues, which are then analyzed separately by assuming specific arrival processes, and in particular the output process of one intersection serves as the input process of an upstream intersection, hence creating the correlation structure that comes with network topologies.

Osorio and Flötteröd (2015) and Osorio et al. (2011) take a different approach for modeling correlated network structure, by extending the deterministic kinematic wave model with additional sources of randomness. Numerical evaluation of the models in Osorio and Flötteröd (2015) and Osorio et al. (2011) allows to investigate the impact of the correlations structure on dynamic build-up, spill back and dissipation of queues. In contrast, our network model does not depart from the deterministic kinematic wave model (that comes with differential equations), but rather builds on the FCTL queue and starts from a model formulation in terms of a multi-dimensional Markov process. Although it is possible to use our Markov network model for numerically calculating transient effects like dynamic build-up and spill back, the present paper develops the mathematical methods for an exact steady-state analysis.

**Outline of the paper.** In Section 2 we provide a detailed model description of the generalized FCTL queue. In Section 3 we present the full analytic solution of the generalized FCTL queue, both in terms of a formal characterization of the probability generating functions of the queue length distribution, and in terms of practically implementable algorithms for calculating the queue length distribution, for any given correlated arrival pattern. In Section 4 we design the network algorithm based on decomposition and the results in Section 3. We also compare our analytical results with extensive discrete-event simulation of the same network model. In Section 5 we present conclusions.

## 2. FCTL queue with correlated arrivals

We now present a generalization of the classical FCTL queue that can deal with correlated arrivals. In Section 2.1 we detail the model and its underlying assumptions. We then discuss in Section 2.2 an example of an arrival pattern that contains some crucial features than are anticipated in network settings. The numerical calculations for that example were performed with the algorithmic method developed in Section 3.

### 2.1. Model description

The first two model assumptions are adopted from the classical FCTL queue van Leeuwaarden (2006):

**Assumption 2.1.** (Discrete-time assumption) The time axis is divided into constant time intervals of unit length, so-called slots, where each slot corresponds to the time needed for a delayed vehicle to depart from the queue. The green and red periods, and thus the cycle time $c$, are assumed to be fixed multiples of one slot. Hence, $g$, $r$, $c$ are integers expressed in slots. Those vehicles that arrive to the queue and are delayed, join the queue at the end of the slot in which they arrive.