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Infinitely many nonnegative solutions for a fractional differential inclusion with oscillatory potential

Yue Yue[1]*, Yu Tian[1]†, Zhanbing Bai[2]‡
1. School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, P.R. China
2. College of Mathematics and System Science, Shandong University of Science and Technology, Qingdao 266590, P.R. China

Abstract: In the present paper we study a fractional differential inclusion with Sturm-Liouville boundary value conditions. By means of nonsmooth critical point theorem, the existence of infinitely many nonnegative solutions is established when the potential F is oscillatory at infinity and origin, respectively.
Keywords: Fractional differential inclusion; Oscillatory assumptions; Multiple solutions; Sturm-Liouville boundary conditions.

1 Introduction and statement of the main results

The purpose of this paper is to discuss the existence of multiple solutions for the fractional differential inclusion problem

$$\begin{cases} -\frac{d}{dt}\left[\frac{1}{2}{}_0D_t^{-\beta}(u'(t)) + \frac{1}{2}{}_tD_T^{-\beta}(u'(t))\right] \in \lambda\partial F(u(t)) \quad \text{a.e. } t\in[0,T], \\ au(0) - b\left[\frac{1}{2}{}_0D_t^{-\beta}u'(0) + \frac{1}{2}{}_tD_T^{-\beta}u'(0)\right] = 0, \\ cu(T) + d\left[\frac{1}{2}{}_0D_t^{-\beta}u'(T) + \frac{1}{2}{}_tD_T^{-\beta}u'(T)\right] = 0, \end{cases} \tag{DI}$$

where ${}_0D_t^{-\beta}$ and ${}_tD_T^{-\beta}$ are the left and right Riemann-Liouville fractional integrals of order $0 \le \beta < 1$, respectively, $a,\ c > 0,\ b,\ d \ge 0$, λ is a positive parameter and ∂F stands for the generalized gradient of a locally Lipschitz function $F : \mathbb{R} \to \mathbb{R}$.

Nonlinear differential equations have always played an important role in the applications of mathematical physics problems, see [25, 26, 27, 28]. The increasing interest in the theory of nonlinear fractional differential equations and inclusions has been mainly motivated by its strong relation with many kinds of science such as Physics, Engineering, Electrochemistry, Porous Media and Control, see for instance [14, 18, 19, 20] and references therein. The applications and extensions of fractional differential equations and inclusions have been investigated by the works [1, 2, 3, 4, 6, 7, 8, 9, 10, 13, 15, 22, 23].

The classical tools as topological degree theory, fixed-point theory and comparison method are applied to study a plenty of fractional differential equations and inclusions, the readers are referred to [12, 21, 29]. We point out that critical-point theory is a classical and matured tool which provides useful means to deal with the boundary value problem of nonlinear fractional differential equations and inclusions. In [15], Jiao and Zhou established variational structure of fractional problem

$$\begin{cases} \frac{d}{dt}\left(\frac{1}{2}{}_0D_t^{-\beta}(u'(t)) + \frac{1}{2}{}_tD_T^{-\beta}(u'(t))\right) + \nabla F(t,u(t)) = 0 \quad a.e.\ t\in[0,T], \\ u(0) = u(T) = 0 \end{cases}$$

and obtained various criteria on the existence of solutions. In [22], Teng and Jia studied the fractional differential inclusion with Dirichlet boundary conditions and proved the existence and multiplicity of solutions by using a variational method based on nonsmooth critical point theory. Furthermore, In [23], Tian and Nieto studied the fractional

*yueyue2017@yeah.net
†Corresponding author: tianyu2992@163.com
‡zhanbingbai@163.com