Applied Mathematics Letters 88 (2019) 111–117

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Limit cycles of a second-order differential equation

Ting Chen[a,b], Jaume Llibre[b,*]

[a] College of Mathematics and Econometrics, Hunan University, Changsha, 410082, PR China
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

ARTICLE INFO

Article history:
Received 20 June 2018
Received in revised form 21 August 2018
Accepted 21 August 2018
Available online 29 August 2018

Keywords:
Mathieu–Duffing type
Limit cycle
Averaging theory

ABSTRACT

We provide an upper bound for the maximum number of limit cycles bifurcating from the periodic solutions of $\ddot{x}+x=0$, when we perturb this system as follows

$$\ddot{x}+\varepsilon(1+\cos^m\theta)Q(x,y)+x=0,$$

where $\varepsilon>0$ is a small parameter, m is an arbitrary non-negative integer, $Q(x,y)$ is a polynomial of degree n and $\theta=\arctan(y/x)$. The main tool used for proving our results is the averaging theory.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

To determine the number of limit cycles of a differential equation is one of the main problems in the qualitative theory of planar differential system. In 1881 Poincaré [1] defined the notion of *limit cycle* of a planar differential system as a periodic orbit isolated in the set of all periodic orbits of the differential system. And he defined the notion of a *center* of a real planar differential system, i.e. of an isolated equilibrium point having a neighborhood filled with periodic orbits. Later on one way to produce limit cycles is by perturbing the periodic orbits of a center, see for instance the papers [2–5] and the references quoted there.

In [6] Mathieu considered the second order differential equation

$$\ddot{x}+b(1+\cos t)x=0, \tag{1}$$

where b is a real constant. It is called Mathieu equation, which is the simplest mathematical model of an excited system depending on a parameter. The more general Ermakov–Pinney equation is the Mathieu–Duffing type equation

$$\ddot{x}+b(1+\cos t)x-x^\beta=0, \tag{2}$$

* Corresponding author.
E-mail addresses: chenting0715@126.com (T. Chen), jllibre@mat.uab.cat (J. Llibre).

https://doi.org/10.1016/j.aml.2018.08.015
0893-9659/© 2018 Elsevier Ltd. All rights reserved.

where β is an integer and $b > 0$. These equations describe the dynamics of a system with harmonic parametric excitation and a nonlinear term corresponding to a restoring force, see the papers [7–10].

We shall study the limit cycles of a kind of generalization of the second-order differential equations (1) and (2). More precisely, the objective of this paper is to consider the second-order differential equations

$$\ddot{x} + \varepsilon(1 + \cos^m \theta)Q(x, y) + x = 0,$$

where $Q(x, y)$ is an arbitrary polynomial of degree n. Eq. (1) is equivalent to the differential system of first order

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x - \varepsilon(1 + \cos^m \theta)Q(x, y). \end{aligned} \tag{3}$$

Many authors are interested in studying the dependence of the number of limit cycles of a differential system with respect to its parameters, and specially on the degree of the polynomials which appear in the system, as for instance in the Hilbert 16th problem see [11–13]. Here our parameters are m and n.

We study the maximum number of limit cycles which can bifurcate from the center of system (3) with $\varepsilon = 0$, where ε is sufficiently small and $\theta = \arctan(y/x)$. More precisely, we consider the planar vector field

$$\chi = \chi(x, y) = (y, -x),$$

and we perturb this vector field χ as follows

$$\chi_\varepsilon = \chi(x, y) + \varepsilon(1 + \cos^m \theta)(0, Q(x, y)).$$

The main result of this paper is the following. For a definition of averaged function of first order see Section 2 and [4].

Theorem 1. *Assume that the average function $f(r)$ of first order associated to the vector field χ_ε is non-zero and $\varepsilon > 0$ sufficiently small.*

(a) *If m is odd the maximum number of limit cycles of χ_ε bifurcating from the periodic solutions of the center χ is at most $n - 1$ using the averaging theory of first order.*
(b) *If m is even the maximum number of limit cycles of χ_ε bifurcating from the periodic solutions of center χ is at most $(n - 1)/2$ or $(n - 2)/2$, when n is odd or even, respectively.*

Moreover these upper bounds are reached.

Theorem 1 is proved in Section 3. Note that the maximum number of limit cycles stated in Theorem 1 depends on the numbers m and n.

We provide a summary about the averaging theory for computing periodic solutions of vector fields that we shall use for proving Theorem 1 in Section 2.

2. Averaging theory for differential systems

In this section we recall some known results of the averaging theory that we shall need for proving Theorem 1. For more details on the averaging theory see [4].

Consider a non-autonomous differential equation of the form

$$\frac{dr}{d\theta} = \chi(r, \theta) = \varepsilon F(r, \theta) + \varepsilon^2 R(r, \theta, \varepsilon), \tag{4}$$

Download English Version:

https://daneshyari.com/en/article/10142475

Download Persian Version:

https://daneshyari.com/article/10142475

Daneshyari.com