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The Nehari manifold of biharmonic equations with p-Laplacian and singular potential

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Abstract

In this paper, we investigate a class of biharmonic equations with p-Laplacian and singular potential as follows:

$$\begin{cases} \Delta^2 u + V_\lambda(x)u + \mathrm{d}iv(\rho(x)\,|\nabla u|^{p-2}\nabla u) = 0 & \text{in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases}$$

where $N \geq 3$, $1 < p < \frac{2N}{N-2}$ except $p = 2$ and $V_\lambda(x) = \lambda a(x) - b(x)$ with $\lambda > 0$. Under some suitable assumptions on a, b and ρ, by using the Nehari manifold, we obtain the existence of nontrivial solutions for λ large enough which improves the existing result in the literature.

Keywords: Biharmonic equations, Nehari manifold, Nontrivial solution, Gagliardo-Nirenberg inequality, Hardy inequality.

1. Introduction

Consider the following biharmonic equations:

$$\begin{cases} \Delta^2 u + V_\lambda(x)u + \mathrm{d}iv(\rho(x)\,|\nabla u|^{p-2}\nabla u) = 0 & \text{in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases} \tag{E_λ}$$

where $N \geq 3, \Delta^2 u = \Delta(\Delta u)$, $1 < p < \frac{2N}{N-2}$ except $p = 2$ and $V_\lambda(x) = \lambda a(x) - b(x)$ with $\lambda > 0$. We assume that the functions a, b and ρ satisfy the following hypotheses:

$(V1)$ $a \in C(\mathbb{R}^N, \mathbb{R}^+)$ and there exists $c_0 > 0$ such that the set $\{a < c_0\} := \{x \in \mathbb{R}^N \mid a(x) < c_0\}$ has finite positive Lebesgue measure;

$(V2)$ $\Omega = \mathrm{int}\{x \in \mathbb{R}^N : a(x) = 0\}$ is nonempty and has smooth boundary with $\bar{\Omega} = \{x \in \mathbb{R}^N : a(x) = 0\}$;

$(V3)$ $b(x)$ is a measurable function on $\mathbb{R}^N$ and there exists $0 < b_0 < \alpha_N^{-1}\left(\frac{N-2}{2}\right)^2$ such that $0 \leq b(x) \leq \frac{b_0}{|x|^2}$ for all $x \in \mathbb{R}^N$, where α_N is defined as in (2.7) below.

$(D1)$ $\rho(x)$ is a sign-changing weight function satisfying $\rho \in L^{2/(2-p)}(\mathbb{R}^N)$ if $1 < p < 2$ and $\rho \in L^\infty(\mathbb{R}^N)$ and $\{\rho > 0\} \cap \Omega$ has finite positive Lebesgue measure if $2 < p < \frac{2N}{N-2}$.

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