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# The Nehari manifold of biharmonic equations with $p$ -Laplacian and singular potential

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## Abstract

In this paper, we investigate a class of biharmonic equations with  $p$ -Laplacian and singular potential as follows:

$$\begin{cases} \Delta^2 u + V_\lambda(x)u + \operatorname{div}(\rho(x)|\nabla u|^{p-2}\nabla u) = 0 & \text{in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases}$$

where  $N \geq 3$ ,  $1 < p < \frac{2N}{N-2}$  except  $p = 2$  and  $V_\lambda(x) = \lambda a(x) - b(x)$  with  $\lambda > 0$ . Under some suitable assumptions on  $a, b$  and  $\rho$ , by using the Nehari manifold, we obtain the existence of nontrivial solutions for  $\lambda$  large enough which improves the existing result in the literature.

*Keywords:* Biharmonic equations, Nehari manifold, Nontrivial solution, Gagliardo-Nirenberg inequality, Hardy inequality.

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## 1. Introduction

Consider the following biharmonic equations:

$$\begin{cases} \Delta^2 u + V_\lambda(x)u + \operatorname{div}(\rho(x)|\nabla u|^{p-2}\nabla u) = 0 & \text{in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases} \quad (E_\lambda)$$

where  $N \geq 3$ ,  $\Delta^2 u = \Delta(\Delta u)$ ,  $1 < p < \frac{2N}{N-2}$  except  $p = 2$  and  $V_\lambda(x) = \lambda a(x) - b(x)$  with  $\lambda > 0$ . We assume that the functions  $a, b$  and  $\rho$  satisfy the following hypotheses:

- (V1)  $a \in C(\mathbb{R}^N, \mathbb{R}^+)$  and there exists  $c_0 > 0$  such that the set  $\{a < c_0\} := \{x \in \mathbb{R}^N \mid a(x) < c_0\}$  has finite positive Lebesgue measure;
- (V2)  $\Omega = \operatorname{int}\{x \in \mathbb{R}^N : a(x) = 0\}$  is nonempty and has smooth boundary with  $\bar{\Omega} = \{x \in \mathbb{R}^N : a(x) = 0\}$ ;
- (V3)  $b(x)$  is a measurable function on  $\mathbb{R}^N$  and there exists  $0 < b_0 < \alpha_N^{-1} \left(\frac{N-2}{2}\right)^2$  such that  $0 \leq b(x) \leq \frac{b_0}{|x|^2}$  for all  $x \in \mathbb{R}^N$ , where  $\alpha_N$  is defined as in (2.7) below.
- (D1)  $\rho(x)$  is a sign-changing weight function satisfying  $\rho \in L^{2/(2-p)}(\mathbb{R}^N)$  if  $1 < p < 2$  and  $\rho \in L^\infty(\mathbb{R}^N)$  and  $\{\rho > 0\} \cap \Omega$  has finite positive Lebesgue measure if  $2 < p < \frac{2N}{N-2}$ .

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