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# Error estimate of second-order finite difference scheme for solving the Riesz space distributed-order diffusion equation

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## Abstract

In the current paper, an error estimate has been proposed to find a second-order finite difference scheme for solving the Riesz space distributed-order diffusion equation. The convergence order of the proposed method is  $\mathcal{O}(\tau^2 + h^2)$ . The numerical results show the efficiency of the new technique.

**Keywords:** Finite difference method, Riesz space distributed-order diffusion equation, unconditional stability, convergence.

**Mathematics Subject Classification:** 65L60, 65L20, 65M70.

## 1 Introduction

Applications of the fractional partial differential equations (PDEs) can be found in [27, 43]. The fractional equations with the distributed-order have been studied by many researchers for example Caputo [4] proposed the application of differential equations with distributed-order derivatives for generalizing stress-strain relations of unelastic media. Also, Caputo in [5, 6] discussed distributed-order time fractional differential equations and distributed-order space fractional differential equations, respectively and derived the solutions with closed form formulae of the classic problems. Authors of [10, 39] gave out diffusion-like equations with distributed-order time and space fractional derivatives for the kinetic description of anomalous diffusion and relaxation phenomena [29]. Author of [26] applied distributed-order diffusion equation to discuss ultraslow and lateral diffusion processes. Also, the applications of fractional equations with space distributed-order can be found in [1, 20, 33, 38, 40].

In the current investigation, we consider the following model [22, 29, 32]

$$\begin{cases} \frac{\partial u(x, t)}{\partial t} = \int_1^2 \varpi(\alpha) \frac{\partial^\alpha u(x, t)}{\partial |x|^\alpha} d\alpha + f(x, t), & x \in \Omega, & 0 < t \leq T, \\ u(x, 0) = u_0(x), & x \in \Omega, \\ u(x, t) = 0, & x \in \partial\Omega, & 0 < t < T, \end{cases} \quad (1.1)$$

in which the Riesz fractional derivative is [21, 37]

$$\frac{\partial^\alpha u(x, t)}{\partial |x|^\alpha} = \frac{-1}{2 \cos(\alpha\pi/2)} \left( {}_x^{RL}\mathfrak{D}_L^\alpha u(x, t) + {}_x^{RL}\mathfrak{D}_R^\alpha u(x, t) \right), \quad (1.2)$$

and also [21, 37]

$${}_x^{RL}\mathfrak{D}_L^\alpha u(x, t) = \frac{1}{\Gamma(2-\alpha)} \frac{d^2}{dx^2} \int_a^x (x-\xi)^{1-\alpha} u(s, t) d\xi, \quad (1.3)$$

$${}_x^{RL}\mathfrak{D}_R^\alpha u(x, t) = \frac{1}{\Gamma(2-\alpha)} \frac{d^2}{dx^2} \int_x^b (\xi-x)^{1-\alpha} u(s, t) d\xi, \quad (1.4)$$

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