Error estimate of second-order finite difference scheme for solving the Riesz space distributed-order diffusion equation

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Error estimate of second-order finite difference scheme for solving the Riesz space distributed-order diffusion equation

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Abstract

In the current paper, an error estimate has been proposed to find a second-order finite difference scheme for solving the Riesz space distributed-order diffusion equation. The convergence order of the proposed method is $\mathcal{O}(\tau^2 + h^2)$. The numerical results show the efficiency of the new technique.

Keywords: Finite difference method, Riesz space distributed-order diffusion equation, unconditional stability, convergence.

Mathematics Subject Classication: 65L60, 65L20, 65M70.

1 Introduction

Applications of the fractional partial differential equations (PDEs) can be found in [27, 43]. The fractional equations with the distributed-order have been studied by many researchers for example Caputo [4] proposed the application of differential equations with distributed-order derivatives for generalizing stress-strain relations of unelastic media. Also, Caputo in [5, 6] discussed distributed-order time fractional differential equations and distributed-order space fractional differential equations, respectively and derived the solutions with closed form formulae of the classic problems. Authors of [10, 39] gave out diffusion-like equations with distributed-order time and space fractional derivatives for the kinetic description of anomalous diffusion and relaxation phenomena [29]. Author of [26] applied distributed-order diffusion equation to discuss ultraslow and lateral diffusion processes. Also, the applications of fractional equations with space distributed-order can be found in [1, 20, 33, 38, 40].

In the current investigation, we consider the following model [22, 29, 32]

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = \displaystyle\int_1^2 \varpi(\alpha)\dfrac{\partial^{\alpha}u(x,t)}{\partial|x|^{\alpha}}d\alpha + f(x,t), \qquad x \in \Omega, \qquad 0 < t \leq T, \\ u(x,0) = u_0(x), \quad x \in \Omega, \\ u(x,t) = 0, \quad x \in \partial\Omega, \quad 0 < t < T, \end{cases} \tag{1.1}$$

in which the Riesz fractional derivative is [21, 37]

$$\frac{\partial^{\alpha}u(x,t)}{\partial|x|^{\alpha}} = \frac{-1}{2\cos(\alpha\pi/2)}\left({}_{x}^{RL}\mathfrak{D}_L^{\alpha}u(x,t) + {}_{x}^{RL}\mathfrak{D}_R^{\alpha}u(x,t)\right), \tag{1.2}$$

and also [21, 37]

$${}_{x}^{RL}\mathfrak{D}_L^{\alpha}u(x,t) = \frac{1}{\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_a^x (x-\xi)^{1-\alpha}u(s,t)d\xi, \tag{1.3}$$

$${}_{x}^{RL}\mathfrak{D}_R^{\alpha}u(x,t) = \frac{1}{\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_x^b (\xi-x)^{1-\alpha}u(s,t)d\xi, \tag{1.4}$$

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