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A mathematical model for the 3d location estimation of 2D echocardiography data

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ABSTRACT

The recovery of 3D left ventricle(LV) shape using 2D echocardiography is very attractable topic in the field of ultrasound imaging. In this paper, we propose a mathematical model to determine the 3D position of LV contours extracted from multiple 2D echocardiography images. We formulate the proposed model as a non-convex constrained minimization problem. To solve it, we propose a proximal alternating minimization algorithm with a solver *OPTI* for quadratically constrained quadratic program. For validating the proposed model, numerical experiments are performed with real ultrasound data. The experimental results show that the proposed model is promising and available for real echocardiography data.

1. Introduction

In company with the emergence of real-time 3D echocardiography(RT3DE), the demands for analysis tools to assess the LV function using RT3DE are steadily increasing [1,2]. However, RT3DE has problems of relatively high cost as well as poor temporal and spatial resolutions compared to 2D echocardiography. For this reason, 2D echocardiography is more preferable in clinical practice despite the usefulness of the RT3DE and hence most of analysis tools are still performed based on the measurements in 2D slices. Recent studies on ultrafast ultrasound imaging technique are expected to improve the poor resolutions of RT3DE and to robustly analyze the LV motion [3,4].

We consider the reconstruction of 3D LV shape using 2D echocardiography. This topic is very attractable because dynamic 3D visualization of the heart is allowed without a 3D imaging scanner. There have been several studies to recover the 3D shape of LV from 2D echocardiography data [5–7]. The main issues are the acquisition of multiple views and their interpolation for representing cardiac chambers in 3D. Thus

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the 3D representation of the LV shape needs the position information of the multiple 2D images. In those studies, additional devices of motor and position sensor have been used for rotating the 2D imaging probe and recording the probe positions, respectively. So, the 3D position and orientation of 2D image planes are spatially pre-determined or tracked using a sensor attached to a probe [8].

This paper focuses on the problem of determining the 3D positions of 2D apical echocardiograms that are generally scanned without depending on additional devices of a motor or a sensor. In our previous work [9], we proposed a mathematical model for determining the 3D position of LV contours extracted in each view. The model was designed by using the fact that the angles between apical long-axis 4, 2 and 3-chamber views are approximately 60° toward each other [10]. Compared to the previous work, the contribution of this paper is to deal with three matters that were not considered before. First, the mathematical model is modified to include the non-zero translation vectors that translate the origin to the inside region of LV in each 2D echocardiogram. Nevertheless, it still can be based on the form $A\mathbf{x} = \mathbf{b}$ with unknown A and $\mathbf{x}$. Second, the proposed model is applied to real LV borders extracted from the real echocardiography. Third, we show that the whole sequence of our proposed algorithm converges to some stationary point based on Kurdyka–Lojasiewicz property; see [11] and references therein.

In order to determine A and $\mathbf{x}$, we consider a non-convex constrained minimization problem. The given minimization problem is a quadratically constrained quadratic program(QCQP) [12] in terms of $\mathbf{x}$ if A is fixed and is also a QCQP in terms of A if $\mathbf{x}$ is fixed. To exploit this fact, we propose a proximal alternating minimization algorithm(PAMA) with a solver [13] for QCQP. That is, we alternately minimize the proximal form [14] of the given model with respect to $\mathbf{x}$ and then with respect to A by using the QCQP solver. We test the proposed model on real LV data. Numerical results show that the proposed model is appropriate.

2. Mathematical model for computing 3D positions

We formulate a mathematical model for finding the 3D positions of the LV contours extracted in the apical long-axis 4, 3 and 2-chamber views abbreviated as A4CH, A3CH and A2CH, respectively.

2.1. A model formulation

Let $\Omega \subseteq \mathbb{R}^2 \times \{0\}$ be an imaging domain. Let $\mathcal{C}_p$, $\mathcal{C}_q$ and $\mathcal{C}_r$ be the sets of LV contour points for the A4CH, A3CH and A2Ch views, defined by $\mathcal{C}_p = \{\mathbf{p}_i \in \Omega \ : \ i = 1, \dots, n\}$, $\mathcal{C}_q = \{\mathbf{q}_i \in \Omega \ : \ i = 1, \dots, n\}$ and $\mathcal{C}_r = \{\mathbf{r}_i \in \Omega \ : \ i = 1, \dots, n\}$, respectively. Let $\mathbf{a} \in \Omega$ be a vector translating the origin to the inner region of LV contour connected by $\mathcal{C}_p$. Likewise, let $\mathbf{b}$ and $\mathbf{c} \in \Omega$ be translation vectors for $\mathcal{C}_q$ and $\mathcal{C}_r$, respectively.

We use notations α, β, ψ and φ to determine the relative positions of the A4CH and A3CH views to the fixed A2CH view. Here, α and ψ mean the angle to rotate around the translated origin in the A4CH view and the angle to rotate about the y-axis, respectively. Similarly, β and φ are defined for the A3CH view.

Next, some additional conditions for contour points located along mitral annulus are given to complete the formulation. Let $\delta_1, \dots, \delta_6$ be circumferential lengths between two adjacent points along the mitral annulus as depicted in Fig. 1. Then we describe the model formulation as follows:

For given $\mathbf{p}_i$, $\mathbf{q}_i$, $\mathbf{r}_i \in \Omega \subseteq \mathbb{R}^2 \times \{0\}$ $(i = 1, \dots, n)$ and $\delta_1, \dots, \delta_6 \in \mathbb{R}^+$, determine the unknowns $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c} \in \mathbb{R}^2 \times \{0\}$, $\mathbf{s}_i$, $\mathbf{t}_i$, $\mathbf{d}_i \in \mathbb{R}^3$ $(i = 1, \dots, n)$ and $\boldsymbol{\Psi}$, $\boldsymbol{\Phi} \in \mathbb{R}^{3\times 3}$ satisfying the following conditions:

$$\boldsymbol{\Psi}^T \boldsymbol{\Psi} = \boldsymbol{\Psi}\boldsymbol{\Psi}^T = \mathbf{I} \quad \text{and} \quad \boldsymbol{\Phi}^T \boldsymbol{\Phi} = \boldsymbol{\Phi}\boldsymbol{\Phi}^T = \mathbf{I}, \tag{1}$$

where $\mathbf{I}$ is the 3×3 identity matrix,

$$\begin{cases} \mathbf{s}_i = \boldsymbol{\Psi}(\mathbf{p}_i - \mathbf{a}), & i = 1, \dots, n, \\ \mathbf{t}_i = \boldsymbol{\Phi}(\mathbf{q}_i - \mathbf{b}), & i = 1, \dots, n, \\ \mathbf{d}_1 = \mathbf{r}_1 - \mathbf{c}, \quad \mathbf{d}_n = \mathbf{r}_n - \mathbf{c}, & \end{cases} \tag{2}$$