

Accepted Manuscript

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PII: S0893-9659(18)30297-0

DOI: <https://doi.org/10.1016/j.aml.2018.08.016>

Reference: AML 5627

To appear in: *Applied Mathematics Letters*

Received date: 25 June 2018

Accepted date: 21 August 2018

Please cite this article as: J. Džurina, S.R. Grace, I. Jadlovská, On nonexistence of Kneser solutions of third-order neutral delay differential equations, *Appl. Math. Lett.* (2018), <https://doi.org/10.1016/j.aml.2018.08.016>

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**ON NONEXISTENCE OF KNESER SOLUTIONS OF THIRD-ORDER
NEUTRAL DELAY DIFFERENTIAL EQUATIONS**

J. DŽURINA, S. R. GRACE, AND I. JADLOVSKÁ

ABSTRACT. The aim of this paper is to complement existing oscillation results for third-order neutral delay differential equations by establishing sufficient conditions for nonexistence of so-called Kneser solutions. Combining newly obtained results with existing ones, we attain oscillation of all solutions of the studied equations.

1. INTRODUCTION

Consider the third-order neutral delay differential equation of the form

$$\left(r_2(r_1y')'\right)'(t) + q(t)x(\sigma(t)) = 0, \quad t \geq t_0 > 0, \quad (1.1)$$

where $y(t) := x(t) + p(t)x(\tau(t))$. Throughout the paper, we will assume that

(H₁) $\sigma, \tau \in C^1([t_0, \infty), \mathbb{R})$, $\sigma(t) < t$, $\tau' \geq \tau_0 > 0$, and $\lim_{t \rightarrow \infty} \tau(t) = \lim_{t \rightarrow \infty} \sigma(t) = \infty$;

(H₂) $p, q \in C([t_0, \infty), [0, \infty))$, $0 \leq p(t) \leq p_0 < \infty$ and q does not vanish identically;

(H₃) $r_1, r_2 \in C([t_0, \infty), (0, \infty))$ satisfy $\int_{t_0}^{\infty} r_1^{-1}(t)dt = \int_{t_0}^{\infty} r_2^{-1}(t)dt = \infty$;

and either

(H_{4a}) $\tau(t) \leq t$ and $\tau \circ \sigma = \sigma \circ \tau$;

or

(H_{4b}) $\tau(t) > t$, $\sigma' > 0$ and $(\sigma^{-1}(t))' \geq \sigma_0 > 0$.

For the sake of brevity, we define the operators

$$L_0y = y, \quad L_1y = r_1y', \quad L_2y = r_2(r_1y')', \quad L_3y = \left(r_2(r_1y')'\right)'.$$

By a solution to equation (1.1), we mean a nontrivial function $x \in C([T_x, \infty), \mathbb{R})$ with $T_x \geq t_0$, which has the property $y, L_1y, L_2y \in C^1([T_x, \infty), \mathbb{R})$, and satisfies (1.1) on $[T_x, \infty)$. We only consider those solutions of (1.1) which exist on some half-line $[T_x, \infty)$ and satisfy the condition $\sup\{|x(t)| : T \leq t < \infty\} > 0$ for any $T \geq T_x$.

As is customary, a solution x of (1.1) is said to be *oscillatory* if it is neither eventually positive nor eventually negative. Otherwise, it is said to be *nonoscillatory*. The equation itself is termed *oscillatory* if all its solutions oscillate.

Following classical results of Kiguradze and Kondrat'ev (see, e.g., [8]), we say that (1.1) has *property A* if any solution x of (1.1) is either oscillatory or satisfies $\lim_{t \rightarrow \infty} x(t) = 0$. Instead of using property A, some authors say that equation is *almost oscillatory*.

To start with, let us state a characterization of possible nonoscillatory, solutions of (1.1). The following result is a modification of the well known Kiguradze lemma [8, Lemma 1.1] based on (H₃).

Lemma 1. *Assume (H₁)–(H₃) and x is a nonoscillatory solution of (1.1). Then there are only two possible classes for y :*

$$N_0 = \{y(t) : (\exists T \geq t_0)(\forall t \geq T)(y(t)L_1y(t) < 0, y(t)L_2y(t) > 0)\}$$

$$N_2 = \{y(t) : (\exists T \geq t_0)(\forall t \geq T)(y(t)L_1y(t) > 0, y(t)L_2y(t) > 0)\}.$$

2010 *Mathematics Subject Classification.* 34C10, 34K11.

Key words and phrases. neutral differential equation, delay, third-order, Kneser solution, oscillation.

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