Applied Ocean Research 80 (2018) 136–147

Contents lists available at ScienceDirect

Applied Ocean Research

journal homepage: www.elsevier.com/locate/apor

Scattering of water waves by an inclined plate in a two-layer fluid

Najnin Islam, R. Gayen*

Department of Mathematics, Indian Institute of Technology, Kharagpur 721302, India

ARTICLE INFO

Keywords:
Two-layer fluid
Wave scattering
Inclined plate
Hypersingular integral equation
Reflection and transmission coefficients
Hydrodynamic force

ABSTRACT

The problem of water wave scattering by an inclined thin plate submerged in the lower layer of a two-layer fluid is examined here using linear theory. In a two-layer fluid, corresponding to a particular frequency wave propagates with two different wavenumbers. Thus we determine the reflection and the transmission coefficients and the hydrodynamic force for both the wavenumbers. This leads to two separate problems. These problems are solved using a hypersingular integral equation method. Several numerical results for the physical quantities varying the inclination, depth and length of the plate are presented. Making suitable adjustment of the parameters published results for a vertical plate submerged in a single layer fluid are recovered.

1. Introduction

Investigation of wave scattering and radiation problems in a two-layer fluid has been a subject of considerable attention. This is because, in reality, an ocean may be considered as a stratified fluid. This stratification which occurs vertically is due to change in temperature or salinity of the water. Very often the change in density is confined to a very thin layer of pycnocline above and below which the density is practically constant. Thus the fluid can be effectively modeled as a two-layer fluid consisting of an upper layer with less density and a lower layer comprised of greater density.

Stokes [1] first analyzed the wave motion in a two-layer fluid. Lamb [2] established that time-harmonic progressive waves can propagate with two wave modes (numbers) – waves with a lower wave number propagate along the free surface whilst those with a greater wave number propagate along the interface. Linton and McIver [3] examined the problem of wave interaction with horizontal cylinders in fluids consisting of a finite upper layer and an infinite lower layer. This problem was extended to three-dimensions by Cadby and Linton [4], who studied wave radiation and diffraction by a sphere submerged in either of the two-layer. Manam and Sahoo [5] studied wave scattering by porous structures in a two-layer fluid using a generalized orthogonal relation. Kashiwagi et al. [6] employed boundary integral equation method to investigate the diffraction problem involving a body of general shape and analyzed the relevant wave induced motion. Bhattacharjee and Sahoo [7] developed an expansion formula and the related mode-coupling relations for studying wave problems in a two-layer fluid with an ice-cover. Using the method of multipoles, Das and Mandal [8] examined the problem of wave radiation by a sphere submerged in a two-layer fluid with an ice-cover. Recently Dhillon et al. [9] have analyzed the problems of surface and interface wave scattering by a thin vertical barrier submerged in the upper fluid of finite depth wherein the lower layer extends infinitely downwards.

Although wave interaction problems involving a vertical barrier submerged either in a single-layer fluid or in a two-layer fluid are well studied, studies involving an inclined plate are not common. Few seminal investigations in this direction are Parsons and Martin [10], Gayen and Mondal [11], Mondal and Banerjea [12]. Maiti and Mandal [13] derived expressions for the reflection and the transmission coefficients in connection with the wave scattering by an inclined plate submerged in deep water. But, they presented the numerical results only for a vertical plate.

Construction of partial breakwaters is important because installation of such breakwaters is less expensive compared to breakwaters that extend from free surface till the bottom of the fluid region. Again to maintain a balance between reflection and transmission and to control these phenomena installation of partial breakwaters is more reasonable. Analysis of wave motion past partial breakwaters in a two-fluid medium may be found in Behera et al. [14,15], Mandal et al. [16].

In the present study, we consider a partial breakwater in the form of a plate of finite width that is inclined to an arbitrary angle to the vertical. To the best of the author's knowledge, there does not exist any research work which examines scattering of surface/interface waves with an inclined plate present in a two-layer fluid. However, inclined plates may serve as effective breakwaters as they penetrate through the layers of water with varying particle velocities and nurture their interactions. This results in deformation of particular orbits which will cause wave breaking and loss of energy in the wave. Also, inclined

* Corresponding author.
E-mail address: rupanwita.gayen@gmail.com (R. Gayen).

https://doi.org/10.1016/j.apor.2018.06.021
Received 2 December 2017; Received in revised form 19 June 2018; Accepted 20 June 2018

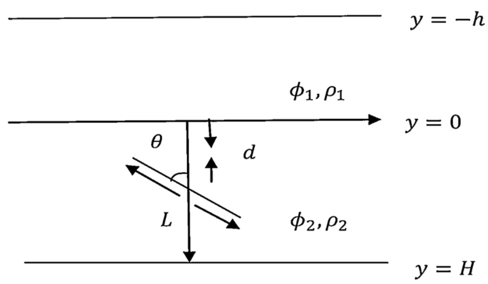

Fig. 1. Definition sketch.

plates serve as a better option as a breakwater for wave attenuation and reduction of wave loads.

In this paper, we deal with the scattering of normally incident waves by a rigid inclined plate submerged in the lower layer of a two-layer fluid. Both the fluid layers are considered to be of finite depth. We apply a hypersingular integral equation method (viz. Parsons and Martin [10,17]) to solve the problem. Solving the integral equations numerically we determine the reflection and the transmission coefficients and the hydrodynamic force acting on the plates. We show that our results for the reflection coefficient completely agree with those for a single-layer fluid when we transform the two-layer fluid to a single-layer fluid by adjusting specific parameters. In addition, we also use the energy identities as a check on the correctness of all numerical results for the reflection and the transmission coefficients. It is observed from the numerical results that the reflection coefficients as well as the wave load on the plate, reduce with an increasing inclination of the plate (Fig. 1).

2. Formulation of the problem

Here we consider a two-layer ocean consisting of two superposed, immiscible, inviscid fluids. The upper fluid is bounded above by a free surface and is of finite depth h. The lower fluid is also of finite depth H. The plane $y = -h$ denotes the rest position of the free surface and the common interface of the two fluids is represented by the horizontal plane $y = 0$, where the y-axis is chosen vertically downwards into the lower fluid and is measured from the undisturbed interface. A thin straight rigid plate L of width $2b$ is inclined at an angle $\theta\left(0 \le \theta \le \frac{\pi}{2}\right)$ to the vertical and is submerged in the lower layer. Let $d(> b\cos\theta)$ be the depth of its mid-point below the undisturbed mean interface of the two fluids. Assuming the fluid motion to be irrotational, the motion in the upper and lower fluids can be described by the velocity potentials $Re\{\phi_1(x, y)e^{-i\sigma\tau}\}$ and $Re\{\phi_2(x, y)e^{-i\sigma\tau}\}$ respectively, where τ denotes the time. The functions $\phi_j(x, y)$ satisfy

$$\nabla^2\phi_j = 0, \text{ in the respective fluid domain.} \tag{2.1}$$

Linearized boundary conditions at the free surface, interface and at the bottom are

$$K\phi_1 + \phi_{1y} = 0 \text{ on } y = -h, \tag{2.2}$$

$$\phi_{1y} = \phi_{2y} \text{ on } y = 0, \tag{2.3}$$

$$\rho(K\phi_1 + \phi_{1y}) = K\phi_2 + \phi_{2y} \text{ on } y = 0, \tag{2.4}$$

$$\phi_{2y} = 0 \text{ on } y = H. \tag{2.5}$$

Here $\rho = \frac{\rho_1}{\rho_2}(\rho_1 < \rho_2)$, ρ_1 and ρ_2 being the densities of the upper and lower layers respectively, $K = \frac{\sigma^2}{g}$; σ is the angular frequency and g is the acceleration due to gravity.

Also, the linearized boundary condition on the fixed barrier is

$$\phi_{2n} = 0 \text{ on } L, \tag{2.6}$$

where the subscript n indicates the differentiation along the normal direction.

In a two-layer fluid, progressive waves are described by

$$\phi_j(x, y) = f_j(u, y)e^{\pm iux}, \text{ in the respective fluid domain.} \tag{2.7}$$

The functions $f_j(u, y)$ $(j = 1, 2)$ are given by

$$f_1\left(u, y\right) = \frac{\sinh uH}{K\cosh uh - u\sinh uh}\left[u\cosh u\left(h + y\right) - K\sinh u\left(h + y\right)\right], f_2\left(u, y\right) = \cosh u\left(H - y\right), \tag{2.8}$$

where u satisfies the dispersion relation

$$\Delta(u) \equiv (1-\rho)u^2 + K^2(\rho + \coth uh\coth uH) - uK(\coth uh + \coth uH) = 0. \tag{2.9}$$

The above dispersion Eq. (2.9) has exactly two real and positive roots, m and M.

The general far field radiation condition is;

$$\phi_j(x, y) = \begin{cases} \sum_{i=1}^{2} A_i f_j(\mu_i, y)e^{\pm i\mu_i x} + \sum_{i=1}^{2} B_i f_j(\mu_i, y)e^{-i\mu_i x} & \text{as } x \to -\infty, \\ \sum_{i=1}^{2} C_i f_j(\mu_i, y)e^{i\mu_i x} & \text{as } x \to \infty, \end{cases} \tag{2.10}$$

where the constants A_i are related to incident waves, so that these are known constants. The constants B_i, C_i are unknown constants to be determined and $\mu_1 = m$, $\mu_2 = M$.

In the present study, we consider the scattering of progressive waves by an inclined plate. It is well established that in a two-layer fluid, for a prescribed frequency, incident waves propagate with two different wavenumbers.

Keeping this in mind, we consider the following problems:

Problem 1: The scattering of an incident wave of wave-number m from the direction of $x = -\infty$.

Problem 2: The scattering of an incident wave of wave-number M from the direction of $x = -\infty$.

In the far field condition (2.10), $A_1 = 1$, $A_2 = 0$ and $A_1 = 0$, $A_2 = 1$ for Problems 1 & 2 respectively. The constants B_1, C_1 (or) B_2, C_2, represent wave amplitude associated with the reflected and the transmitted waves in surface and internal modes.

Therefore, the far-field forms of $\phi_j (j = 1, 2)$ for an incident wave of wavenumber m can be represented as

$$\phi_j(x, y) = \begin{cases} \phi_{jm}^{inc}(x, y) + r_{-}\phi_{jm}^{inc}(-x, y) + R_{-}f_j(M, y)e^{-iMx} & \text{as } x \to -\infty, \\ r_{+}\phi_{jm}^{inc}(x, y) + R_{+}f_j(M, y)e^{iMx} & \text{as } x \to \infty, \end{cases} \tag{2.11}$$

where

$$\phi_{jm}^{inc}(x, y) = f_j(m, y)e^{imx}, \tag{2.12}$$

and the far-field forms of ϕ_j for the incident wave of wavenumber M can be represented as

$$\phi_j(x, y) = \begin{cases} \phi_{jM}^{inc}(x, y) + \hat{r}_{-}f_j(m, y)e^{-imx} + \hat{R}_{-}\phi_{jM}^{inc}(-x, y) & \text{as } x \to -\infty, \\ \hat{r}_{+}f_j(m, y)e^{imx} + \hat{R}_{+}\phi_{jM}^{inc}(x, y) & \text{as } x \to \infty, \end{cases} \tag{2.13}$$

where

$$\phi_{jM}^{inc}(x, y) = f_j(M, y)e^{iMx}. \tag{2.14}$$