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Pseudo-impulsive solutions of the forward-speed diffraction problem using a high-order finite-difference method

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ABSTRACT

This paper considers pseudo-impulsive numerical solutions to the forward-speed diffraction problem, as derived from classical linearized potential flow theory. Both head- and following-seas cases are treated. Fourth-order finite-difference approximations are applied on overlapping, boundary-fitted grids to obtain solutions using both the Neumann-Kelvin and the double-body flow linearizations of the problem. A method for computing the pseudo-impulsive incident wave forcing in finite water depth using the Fast Fourier Transform (FFT) is presented. The pseudo-impulsive scattering solution is then Fourier transformed into the frequency domain to obtain the wave excitation forces and the body motion response. The calculations are validated against reference solutions for a submerged circular cylinder and a submerged sphere. Calculations are also made for a modern bulk carrier, showing good agreement with experimental measurements.

1. Introduction

Accurately predicting the wave-induced loading and response of sailing ships is important to ensure their safety and reliability. While high-fidelity numerical methods based on solving the Navier-Stokes equations (CFD) are now feasible for simulating short-term individual events, they are still too computationally expensive for routine analysis and preliminary design optimization, see for example [1–6]. Thus, methods based on the assumptions of a linearized potential flow are still heavily used to map out the complete response spectrum, guide the initial design process and set up extreme loading scenarios for more refined CFD analysis.

The most efficient linearized potential flow solutions to this problem are obtained using two-dimensional (2D) strip-type methods, for example based on [7] or [8]. Despite the theoretical weaknesses inherent to these methods, they generally produce excellent results for integrated quantities like global loading and motion response, although detailed local quantities are not available. Three-dimensional (3D) methods are also widely used, most commonly based on the *boundary element* method (BEM) and the free-space (Rankine) Green function e.g. [9–14]. A smaller number of time-domain solutions using the free-surface Green function have also been developed, for example [15–18]. *Finite element* models for wave structure interaction problems can also be found, for example [19,20]. Two well-known cases of the use of *finite difference* method for the ship wave resistance problem are [21,22].

Motivated by the difficulties faced by BEM methods in obtaining a linear scaling of the solution effort with increasing resolution, we have been developing a high-order finite difference framework for nonlinear water wave simulation and wave-structure interaction, see for example [23–29] and [30]. While the total number of unknowns required here is typically an order of magnitude larger than that required by BEM methods, the resulting system matrix is sparse, leading to a linear scaling of the solution effort, as demonstrated for example by [26]. This is especially attractive for computing second- and higher-order wave forces (for example added resistance [31]), where convergence of the solution typically requires high resolution.

In this paper, we present our solution to the forward-speed diffraction problem as implemented in the above described high-order finite-difference framework. Either the Neumann-Kelvin or the double-body flow linearization may be adopted. Inspired by the work of [15], solutions are obtained in the time-domain using a pseudo-impulsive incident wave which is tuned to include only a limited range of frequencies. The pseudo-impulsive incident wave potential and its derivatives can be computed from elementary functions in infinite water depth, but in finite water depth no simple closed-form representation is available. In this case, we compute the wave kinematics using Fast Fourier Transforms, a fast and accurate method which has not yet appeared in the literature (to our knowledge).

A *method of lines* approach [32] is applied to obtain the discrete solution, with the explicit fourth-order Runge-Kutta scheme chosen for

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the time-integration. For the Laplace problem, fourth-order spatial finite difference schemes are developed on overlapping, boundary-fitted grids, resulting in a consistently fourth-order accurate solution, as demonstrated by [26]. Frequency-domain quantities are obtained by taking the Fourier transform of the time-domain results, and forces are computed by integrating the pressure over the surface of the body (using at least fourth-order accurate integration schemes).

The implementation supports waves incident from both ahead of the beam ($\pi/2 \le \beta \le 3\pi/2$, *head seas*), and from abaft the beam ($0 \le \beta < \pi/2$ or $3\pi/2 < \beta \le 2\pi$, *following seas*). Since the solution is obtained in a moving frame of reference fixed to the mean position of the ship, the following seas problem is complicated by the non-unique relationship between encounter frequency ω_e and wave frequency ω_0. Following, for example [17,33], we solve three distinct pseudo-impulsive diffraction problems in this case. Port/starboard symmetry of the ship geometry can also be exploited to solve on only half of the physical domain.

Validation results are presented using both semi-analytical solutions and experimental measurements. First the forward-speed wave excitation forces on both a submerged circular cylinder and a submerged sphere are shown to converge towards the corresponding semi-analytical results. Then a modern bulk carrier is analyzed in both head and following seas, and both wave forces and body motions are shown to compar every well with experimental measurements and other numerical calculations. Finally we note that the model described here, *OceanWave3D-Seakeeping* [34], is available as an open-source code.

2. Formulation of the problem

The details of the classic mathematical formulation for ship motions, including the diffraction problem, can be found for example in [35]. In this section, we include a brief review of the theory and illustrate the details of our solution. The focus here is on the diffraction problem, while the details of our solution to the *radiation* problem can be found in [26].

A coordinate system $O - xyz$ is considered which is attached to the mean position of the vessel and moves steadily with the same forward speed U. The incident wave makes an angle β with the positive x-axis, and is scattered by the moving vessel which has a steady forward speed U. See also Fig. 1. Note that except for the steady forward motion, the vessel is assumed to be stationary in the diffraction problem. For the radiation problem, the body has (at least) six degrees of freedom denoted by: surge ξ_1, sway ξ_2, heave ξ_3, roll ξ_4, pitch ξ_5 and yaw ξ_6, which are also shown in Fig. 1.

2.1. Governing equations

The flow domain is bounded by the free surface S_f, the surface of the body S_b, the sea bed S_d and the far-field truncation boundary S_∞ which is required to limit the extent of the computational domain. Here S_f and S_b are the mean undisturbed free- and body-surfaces respectively. Assuming potential flow theory, all components of the flow velocity are defined by a velocity potential $\phi_s(x, y, z, t)$. This potential describes the flow due to the unknown scattered waves. The continuity condition can be specified by the Laplace equation as follows:

$$\nabla^2\phi_s = \frac{\partial^2\phi_s}{\partial x^2} + \frac{\partial^2\phi_s}{\partial y^2} + \frac{\partial^2\phi_s}{\partial z^2} = 0. \tag{2.1}$$

Note that the total velocity potential in the moving frame of reference can be decomposed as follows:

$$\overline{\phi}_s = -Ux + \phi_b' + \phi_s. \tag{2.2}$$

The *base flow* is shown here by ϕ_b', and for the Neumann-Kelvin linearization $\phi_b' = 0$. For the case of the *double-body* linearization, the base flow can be obtained by solving the following steady-flow problem ($\phi_b' = \phi_{\rm db}$):

$$\nabla^2\phi_{\rm db} = 0, \tag{2.3}$$

$$\frac{\partial\phi_{\rm db}}{\partial n} = \boldsymbol{W}\cdot\boldsymbol{n} \quad \text{on } S_b, \tag{2.4}$$

$$\frac{\partial\phi_{\rm db}}{\partial z} = 0 \quad \text{on } z = 0, \tag{2.5}$$

$$\nabla\phi_{\rm db} \to 0 \quad \text{in the far field } (S_\infty), \tag{2.6}$$

where $\boldsymbol{W} = (U, 0, 0)$ and $\boldsymbol{n}$ is the unit normal vector to the body surface, directed into the body (out of the fluid). This boundary value problem describes an infinite-domain potential flow around a combination of the body and its mirror image with respect to the $z = 0$ plane.

2.2. Boundary conditions

The free surface S_f is subject to the linear dynamic and kinematic conditions as follows:

$$\frac{\partial\phi_s}{\partial t} = -g\zeta + U\frac{\partial\phi_s}{\partial x} - \nabla\phi_b'\cdot\nabla\phi_s \quad \text{at } z = 0, \tag{2.7}$$

$$\frac{\partial\zeta}{\partial t} = \frac{\partial\phi_s}{\partial z} + U\frac{\partial\zeta}{\partial x} - \nabla\phi_b'\cdot\nabla\zeta + \zeta\frac{\partial^2\phi_b'}{\partial z^2} \quad \text{at } z = 0. \tag{2.8}$$

Here g is the acceleration due to gravity and ζ defines the free-surface elevation of the scattered waves. At the sea bed S_d and the far-field truncation boundary S_∞ the following no-flux condition is applied:

$$\boldsymbol{n}\cdot\nabla\phi_s = 0. \tag{2.9}$$

At the surface of the body S_b the similar no-flux condition is applied to the flow field comprised of the incident wave $\phi_0(x, y, z, t)$ and the scattered wave $\phi_s(x, y, z, t)$:

$$\boldsymbol{n}\cdot\nabla(\phi_0 + \phi_s) = 0 \Rightarrow \boldsymbol{n}\cdot\nabla\phi_s = -\boldsymbol{n}\cdot\nabla\phi_0. \tag{2.10}$$

The numerical solution to this initial boundary value problem is described in Section 3.

2.3. Implementation of the body boundary condition

In order to obtain the scattering velocity potential, it is required to know the body boundary condition in the time domain. To this end, first the velocity potential of a linear monochromatic incident wave is considered, which can be defined everywhere in the constant-depth domain by:

$$\phi_0(\mathbf{r}, t) = \mathrm{Re}\left\{\frac{\mathrm{i}A\,g}{\omega_0}\frac{\cosh k(z+h)}{\cosh kh}e^{-\mathrm{i}k\alpha}e^{\mathrm{i}\omega_e t}\right\}, \tag{2.11}$$

in which $\mathbf{r} = (x, y, z)$ is the position vector, and A is the amplitude of the incident wave, which without loss of generality is assumed to be unity. Here ω_0 is the wave frequency and ω_e is the encounter frequency, and they are related through:

$$\omega_e = \omega_0 - kU\cos\beta, \tag{2.12}$$

where $k = 2\pi/\lambda$ is the wave number defined for the wave with length λ. The heading angle is β and is measured with respect to the positive x-axis. The water depth is given by h, and α is a phase function defined as:

$$\alpha = x\cos\beta + y\sin\beta. \tag{2.13}$$

Note that with the above definition, head-seas and following-seas conditions are given by $\beta = \pi$ and $\beta = 0$ respectively. Moreover the incident wave elevation will be:

$$\zeta_0(\mathbf{r}, t) = \mathrm{Re}\{e^{-\mathrm{i}k\alpha}e^{\mathrm{i}\omega_e t}\}. \tag{2.14}$$

If the interest was to find the scattering velocity potential for a single incident wave with a specific frequency, then Eq. (2.11) would be