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Generalized conformal Hamiltonian dynamics and the pattern formation equations

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Abstract

We demonstrate the significance of the Jacobi last multiplier in Hamiltonian theory by explicitly constructing the Hamiltonians of certain well known first-order systems of differential equations arising in the activator-inhibitor (AI) systems. We investigate the generalized Hamiltonian dynamics of the AI systems of Turing pattern formation problems, and demonstrate that various subsystems of AI, depending on the choices of parameters, are described either by conformal or contact Hamiltonian dynamics or both. Both these dynamics are subclasses of another dynamics, known as Jacobi mechanics. Furthermore we show that for non Turing pattern formation, like the Gray-Scott model, may actually be described by generalized conformal Hamiltonian dynamics using two Hamiltonians. Finally, we construct a locally defined dissipative Hamiltonian generating function [13] of the original system. This generating function coincides with the "free energy" of the associated system if it is a pure conformal class. Examples of pattern formation equation are presented to illustrate the method.

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