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On cohomology of filiform Lie superalgebras

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Abstract: Suppose the ground field $\mathbb{F}$ is an algebraically closed field of characteristic different from 2, 3. We determine the Betti numbers and make a decomposition of the associative superalgebra of the cohomology for the model filiform Lie superalgebra. We also describe the associative superalgebra structures of the (divided power) cohomology for some low-dimensional filiform Lie superalgebras.

Keywords: filiform Lie superalgebra; Betti number; associative superalgebra

Mathematics Subject Classification 2010: 17B30, 17B56

0 Introduction

In 1970, in the study of the reducibility of the varieties of nilpotent Lie algebras, Vergne introduced the concept of filiform Lie algebras and showed the fact that every filiform Lie algebra can be obtained by an infinitesimal deformation of the model filiform Lie algebra L_n (see [1]). Since then, the study of the filiform Lie algebras, especially the model filiform Lie algebra, has become an important subject. Many conclusions on cohomology of the model filiform Lie algebra with coefficients in the trivial module have been obtained. For example, the Betti numbers for L_n with coefficients in the trivial module over a field of characteristic zero have been calculated in [2–4]. A result, in [5], states that the filiform Lie algebras L_n and $\mathfrak{m}_2(n)$ have the same Betti numbers over a field of characteristic two, which is different from the case of characteristic zero. Moreover, the first three Betti numbers of L_n and $\mathfrak{m}_2(n)$ over $\mathbb{Z}_2$ have been calculated in [6]. As what happens in the Lie case, every filiform Lie superalgebra can be obtained by an infinitesimal deformation of the model filiform Lie superalgebra $L_{n,m}$. Many conclusions on cohomology of the model filiform Lie superalgebra with coefficients in the adjoint module have been obtained. For example, Khakimdjanov and Navarro gave a complete description of the second cohomology of $L_{n,m}$ with coefficients in the adjoint module in [7–11]. The first cohomology of $L_{n,m}$ with coefficients in the adjoint module has been described in [12] by calculating the derivations. However, in the trivial module case, less of work is done for $L_{n,m}$.

Throughout this paper, the ground field $\mathbb{F}$ is an algebraically closed field of characteristic different from 2, 3 and all vector spaces, algebras are over $\mathbb{F}$. In the characteristic zero case, for any non-negative integer k, we make a decomposition of $\mathrm{H}^k(L_{n,m})$ by the Hochschild-Serre spectral sequences, moreover, we can describe completely the Betti number of $\mathrm{H}^\bullet(L_{n,m})$ and the superalgebra structures of the cohomology for some

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