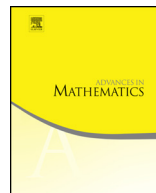




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Quasi-elliptic cohomology I

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ABSTRACT

Quasi-elliptic cohomology is a variant of elliptic cohomology theories. It is the orbifold K-theory of a space of constant loops. For global quotient orbifolds, it can be expressed in terms of equivariant K-theories. Thus, the constructions on it can be made in a neat way. This theory reflects the geometric nature of the Tate curve. In this paper we provide a systematic introduction of its construction and definition.

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1. Introduction

An elliptic cohomology theory is an even periodic multiplicative generalized cohomology theory whose associated formal group is the formal completion of an elliptic curve. The elliptic cohomology theories form a sheaf of cohomology theories over the moduli stack of elliptic curves $\mathcal{M}_{ell}$. Tate K-theory over $\mathrm{Spec}\mathbb{Z}((q))$ is obtained when we restrict it to a punctured completed neighborhood of the cusp at ∞ , i.e. the Tate curve $Tate(q)$ over $\mathrm{Spec}\mathbb{Z}((q))$ [Section 2.6, [2]]. The relation between Tate K-theory and string theory

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is better understood than most known elliptic cohomology theories. In addition, Tate K-theory has the closest ties to Witten's original insight that the elliptic cohomology of a space X is related to the $\mathbb{T}$ -equivariant K-theory of the free loop space $LX = \mathbb{C}^\infty(S^1, X)$ with the circle $\mathbb{T}$ acting on LX by rotating loops. Ganter gave a careful interpretation in Section 2, [6] of this statement that the definition of G -equivariant Tate K-theory for finite groups G is modelled on the loop space of a global quotient orbifold.

Other than the theory over $\mathrm{Spec}\mathbb{Z}((q))$, we can define variants of Tate K-theory over $\mathrm{Spec}\mathbb{Z}[q]$ and $\mathrm{Spec}\mathbb{Z}[q^\pm]$ respectively. The theory over $\mathrm{Spec}\mathbb{Z}[q^\pm]$ is of especial interest. Inverting q allows us to define a sufficiently non-naive equivariant cohomology theory and to interpret some constructions more easily in terms of extensions of groups over the circle. The resulting cohomology theory is called quasi-elliptic cohomology. Its relation with Tate K-theory is

$$QEll_G^*(X) \otimes_{\mathbb{Z}[q^\pm]} \mathbb{Z}((q)) = (K_{Tate}^*)_G(X) \quad (1.1)$$

which also reflects the geometric nature of the Tate curve. As discussed in Remark 3.20, $QEll_{\mathbb{T}}^*(\mathrm{pt})$ has a direct interpretation in terms of the Katz–Mazur group scheme T [Section 8.7, [14]]. The idea of quasi-elliptic cohomology is motivated by Ganter's construction of Tate K-theory [5]. It is not an elliptic cohomology but a more robust and algebraically simpler treatment of Tate K-theory. This new theory can be interpreted in a neat form by equivariant K-theories. Some formulations in it can be generalized to equivariant cohomology theories other than Tate K-theory.

Via quasi-elliptic cohomology theory, we show in this paper that G -equivariant Tate K-theory for any compact Lie group G is given by the $\mathbb{T}$ -equivariant K-theory of the ghost loops [Section 2.4], or constant loops [Section 2.3] inside the free loop space LX . Moreover, as shown in Section 4.1, quasi-elliptic cohomology can be defined not only for G -spaces but also for orbifolds. Applying the same idea, we obtain a loop construction for orbifold Tate K-theory via orbifold quasi-elliptic cohomology theory.

This paper aims to provide a reference for this elegant theory and a systematic introduction of its construction and definition. In Section 2, for any compact Lie group G , we construct G -equivariant quasi-elliptic cohomology from a loop space via bibundles. Thus, we in fact give a construction by loop space of G -equivariant Tate K-theory for compact Lie groups G . In Section 2 [11] we showed the construction when G is a finite group, which, as shown in Section 2, can be generalized to the case when G is a compact Lie group. We discuss the subtle points of this generalization in Section 2.3. In Section 3 we give the definition of quasi-elliptic cohomology $QEll_G^*(-)$ with G a compact Lie group, set up the theory and show its properties. We gave a different definition of $QEll_G^*(-)$ with G a finite group in Definition 3.10, [11], which is equivalent to the definition in this paper. In Section 4, we present the construction of orbifold quasi-elliptic cohomology via the loop space of bibundles. Moreover, we give another construction motivated by Ganter's construction of orbifold Tate K-theory in [7]. The two constructions of orbifold quasi-elliptic cohomology are equivalent.

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