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Embeddings for the Jordan algebra of a bilinear form

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Let K be a field of characteristic zero and let J be a Jordan algebra with a formal trace. We prove that the algebra J can be embedded into a Jordan algebra of a non-degenerate symmetric bilinear form over some associative and commutative K-algebra C if and only if J satisfies all trace identities of the Jordan algebra of a non-degenerate symmetric bilinear form over the field K. This is an extension of results of Procesi and Berele concerning the analogous problem for the associative matrix algebras and the matrix algebras with involution. As a consequence of these results we also prove that the ideal of all trace identities of the Jordan algebra of a non-degenerate symmetric bilinear form over K satisfies the Specht property.

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Polynomial identities
Jordan algebras

1. Introduction

The development of the theory of algebras satisfying polynomial identities (PI algebras for short) was initiated aiming at the following important general problem. What can one say about the structure of an algebra knowing it satisfies a polynomial identity? We will not recollect here the main structure theorems concerning PI algebras but instead we refer the reader to the monographs [9, Sections 1.11, 1.12] and [19, Chapter 1] for an extensive treatment of the topic.

A very important question arises. Given a ring (or an algebra) that satisfies polynomial identities what can one say about its subrings (subalgebras)?

Freely restating the latter question, assume R is, in some sense, a "good" PI ring, and we want to embed some ring S into R. Then one may look for necessary and/or sufficient conditions that guarantee such an embedding. Such problems are classical in PI theory, see for example the papers by Malcev [22–24] and by Bokut [5]. The matrix rings are "good" ones: they are quite well understood, and their importance in Ring theory is enormous without any doubt. Thus describing conditions for embedding a ring into the $n \times n$ matrices must be of importance. One has an immediate necessary condition for embedding a ring S into the $n \times n$ matrices: S must satisfy all polynomial identities of the $n \times n$ matrices. This condition though turned out not sufficient. In the seventies, Amitsur [1] and Small [34], gave independently examples of rings satisfying all identities of $n \times n$ matrices over a field but not embeddable into matrices of order n over any commutative domain.

A satisfactory answer to the embedding problem in general is not known yet. On the other hand Procesi [28] proved that the answer is positive in the variety of algebras with trace satisfying the Cayley–Hamilton identity of degree n, over a field of characteristic zero. Here we recall that the Cayley–Hamilton polynomial of a matrix a can be written as a polynomial whose coefficients are polynomials in the traces of a^k, $k \geq 1$. Moreover according to the celebrated theorem proved independently by Razmyslov [29], and by Procesi [27], all trace identities of the $n \times n$ matrices follow from the Cayley–Hamilton polynomial of degree n. It follows that an associative algebra with trace admits a trace preserving embedding into $n \times n$ matrices over some commutative algebra if and only if it satisfies all trace identities of the $n \times n$ matrices.

The above discussion shows that it is natural to study the embedding problem in other varieties of algebras, having "richer" signature. In 1990, Berele [4] showed that if an algebra with trace and involution satisfies the same $*$-trace identities as the $n \times n$ matrices with (symplectic or transpose) involution, then it has a trace- and involution-preserving embedding into $n \times n$ matrices, thus generalizing Procesi's theorem. The methods used by Procesi and Berele rely on the existence of certain universal map satisfied by matrix algebras. Such a map was introduced and studied by Amitsur, see [2]. The results