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Shrinkage-free tension stiffening law for various concrete grades

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HIGHLIGHTS

- The shrinkage effect drastically changes the shape of tension stiffening relations.
- A shrinkage-free tension stiffening law for reinforced concrete ties was proposed.
- An extensive database (108 members) ensures the universal applicability of the law.
- The prediction results were validated against independent test data.

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ABSTRACT

The present study proposes a new tension stiffening law for reinforced concrete (RC) that takes into account the shrinkage effect occurring prior to the external loading. Due to the restraining action of reinforcement, shrinkage induces tension stresses in the concrete which may significantly reduce the crack resistance and increase deformations of the RC member. The proposed tension stiffening model is based on the test data of seven experimental programs reported in the literature, including 108 RC tension members covering a wide range of concrete grade, reinforcement ratio and bar diameter. The study has shown that shrinkage drastically changes the shape of tension stiffening relations with reinforcement ratio being the most important parameter responsible for this effect. This study reports a limited validation analysis of the proposed constitutive law based on experimental data reported herein. For that tests on four tensile RC members with measured free shrinkage strain have been carried out. The comparative analysis has shown good agreement between the experimental and predicted load–strain and tension stiffening relations.

1. Introduction

Reinforced concrete (RC) is an exceptional structural material due to two aspects: the extent of its practical use and the complexity of its mechanical behaviour. The latter characterisation is attributed to such phenomena as concrete shrinkage and creep as well as cracking, tension softening and tension stiffening, all being highly interrelated with each other. Tension softening is a property of plain concrete to transmit tensile stresses in the cracked section whereas tension stiffening is the ability of concrete to carry tensile stresses in the sections between cracks due to the bond action with reinforcement. Tension stiffening parameters have a significant effect on numerical results of deformation and crack analysis [1,2] of RC structures.

Recently, a new concept of crack analysis of reinforced concrete members has been proposed in [3,4]. The philosophy behind the proposed methodology is to establish mean spacing between primary cracks through the compatibility of the stress-transfer and mean deformation approaches. Parameters of crack spacing are obtained by equating the mean strains of the tensile reinforcement defined by these approaches. The technique considers a single RC block of a length of the mean crack spacing assuming that it represents the averaged deformation behaviour of a cracked member. Based on the experimental evidence, the reinforcement strain within the block is characterized by a strain profile consisting of straight lines. The model was shown to be a simple and mechanically sound tool for predicting mean crack spacing of RC members. However, a comparative analysis of the predictions to the test data has shown that the accuracy of the proposed approach of crack analysis is strongly dependent on the adequacy of the assumed tension stiffening model as it significantly affects the deformations in the tensile reinforcement [4].

A number of approaches have been proposed to take into account the tension stiffening effect in the serviceability analysis of RC structures [5–13]. The *stress-transfer* approach based on the

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bond-slip interaction between concrete and reinforcement most realistically deals with discrete cracking phenomenon [3,6,9,11]. However, lack of adequate bond-slip models and complex mechanisms of analysis limit a wider application of the latter approach. Models with *smeared* (averaged) representation of deformations and cracks due to their simplicity are most extensively used in the numerical applications [5,7–10,12]. In the *smeared crack* approach, tension stiffening can be attributed either to concrete [7,8,12] or reinforcement [5,10]. Gribniak et al. [12] has proposed a stochastic approach in assessing tension stiffening. A simplified approach suggested by Eurocode 2 [13] based on the interpolation formula relating non-cracked and fully cracked states is frequently applied for the deformation analysis of RC ties. However, a statistical analysis [14] has shown that Eurocode 2 provides a far too stiff deformation response, particularly for lightly reinforced members. The prediction errors at various load levels ranged from 25 to 61% for lightly reinforced tensile members (ρ <1.6%) and from 14 to 30% for the members with larger amounts of reinforcement ($\rho \geq$1.6%). As was shown in [14], the above inaccuracies to a significant extent were due to the shrinkage effect occurring prior to loading that was not taken into account by the Eurocode 2 [13]. Due to the restraining action of reinforcement or supports, shrinkage induces tension stresses in the concrete, which might significantly reduce crack resistance and increase deformations of the member [15]. Investigations [16–21] have shown that shrinkage may indeed have a significant effect on deformations of RC members subjected to short-term loading. However, very few tension stiffening models were proposed [16] that take into account or remove the shrinkage effect before calibrating the model; moreover, such models were generally based on a limited amount of test data. Recent investigations of tension stiffening [22–26] were mainly dedicated to new types of concrete and reinforcement.

The present study aims at proposing a new tension stiffening law being derived from a large amount of test data reported in the literature. The test data covers a wide range of geometrical and material properties such as concrete grade, reinforcement ratio and bar diameter. The proposed tension stiffening relationship has the removed shrinkage effect occurring prior to the external loading. The study reports a limited independent validation analysis of the proposed constitutive law based on experimental data reported herein.

2. Test data employed for deriving tension stiffening model

The new model is based on the test data of 7 experimental programs [27–33], listed in Table 1, which involved 108 RC elements (2800 measurements) having different concrete compressive strength up to 70 MPa. In addition to the compressive strength of concrete, the experimental programs covered a wide range of geometrical characteristics such reinforcement ratio and diameter of reinforcement bars. All the experimental programs involved prismatic specimens with nominally square sections reinforced by a single bar subjected to short-term axial tension. The specimens were tested either by controlling the deformations, as adopted in programs No. 1–5, or alternatively, controlling the applied tensile force, as adopted in programs No. 6 and 7.

The main characteristics of the specimens are given in Table 1. The first four columns refer to the test program number, the literature source of the program, the numbers of the tested elements, and the number of measurements in this program, respectively. Further parameters in Table 1 are: the actual height (h) and width (b) of the section; the concrete cover (c); the length of the specimen (L); the diameter of reinforcement bars (D); the area of reinforcement (A_s); the reinforcement ratio (ρ); the cylinder ($\varnothing 150 \times 300$ mm) strength (f_c); and the shrinkage strain (ε_{cs}) measured at the age of testing. When the values of the parameters varied within a range, the range of values rather than individual values are stated in the table. In the cases when the experimental shrinkage strain ε_{cs} was not reported, it was assessed by the Eurocode 2 provisions using available test characteristics responsible for shrinkage.

The material characteristics needed for the analysis such as concrete tensile strength and modulus of elasticity were defined at the time of the short-term tests by the Eurocode 2 provisions based on the compressive strength:

$$f_{ct} = 0.3 \cdot (f_c - 8)^{(2/3)} \text{ when } f_c \leq 58 \text{ MPa} \tag{1a}$$

$$f_{ct} = 2.12 \cdot \ln(1 + (f_c/10)) \text{ when } f_c > 58 \text{ MPa} \tag{1b}$$

$$E_c = 22000 \cdot (f_c/10)^{0.3} \text{ [MPa]} \tag{2}$$

3. Basic equations for deriving tension stiffening stresses

The current analysis is based on smeared crack concept where stress in concrete is taken as the average tensile stress due to tension softening and the bond action between concrete and reinforcement bar, herein collectively called the tension stiffening. Based on the strain compatibility, it is assumed that

$$\varepsilon_m = \varepsilon_s = \varepsilon_{ct} \tag{3}$$

where $\varepsilon_m, \varepsilon_s$ and ε_{ct} are the mean strains of the RC member, reinforcement and concrete, respectively.

The tension stiffening stress–strain relations can be obtained from load–strain diagrams of the experimental specimens (Table 1). Based on the equilibrium of internal forces and the external load,

$$P = N_s + N_c \tag{4}$$

where the internal forces acting in reinforcement, N_s, and concrete, N_c, are assessed as follows:

$$N_s = \varepsilon_s A_s E_s \tag{5}$$

$$N_c = \sigma_{ct} A_c \tag{6}$$

Then from Eqs. (3)–(6), the average tensile stress in concrete can be expressed as:

Table 1
Main characteristics of the test specimens used for the constitutive modelling.

No.	Reference	No. of elements	n	h	b	c	L	D	A_s	ρ	f_c	ε_{cs}
				mm					mm^2	%	MPa	μm/m
1	Farra and Jaccoud (1993)	1–94	2184	100	100	40–45	1150	10–20	79–314	0.8–3.2	35.4–68.8	–
2	Wu and Gilbert (2008)	95–98	264	100	100	42–44	1100	12–16	113–201	1.1–2.1	21.6–24.7	28–249
3	Choi and Maekawa (2003)	99–102	106	100	100	42	1470	16	201	2.1	35.1–40.5	–
4	Noghabai (2000)	103–104	91	80–112	80–112	32–48	960	16	201	1.6–3.2	45.6	–
5	Stroband (1991)	105–106	69	100	100	42–44	935	12–16	113–201	1.1–2.1	49.6	–
6	Scott and Gill (1987)	107	51	103	101	46	1500	12	86	0.8	36.0	–
7	Lorrain et al. (1998)	108	35	100	100	44	2000	12	113	1.1	42.0	–