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A correlation coefficient for belief functions ☆

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ABSTRACT

How to manage conflict is still an open issue in Dempster–Shafer (D-S) evidence theory. The conflict coefficient k in D-S evidence theory cannot represent conflict reasonably, especially sometimes two Basic Probability Assignments (BPAs) are identical but k is not zero. Jousselme distance can well measure the similarity or conflict between two BPAs but it becomes invalid in some cases. Some scholars introduced the concept of correlation coefficient to measure the similarity of two BPAs. However, existing correlation coefficients of BPAs are unstable or insensitive to quantify the conflict and sometimes deduce wrong results. In this paper, a novel correlation coefficient is proposed which takes into consideration both the non-intersection and the difference among the focal elements. It satisfies all the requirements for a metric. Some numerical examples and comparisons of this paper demonstrate the effectiveness of the correlation coefficient.

1. Introduction

Dempster–Shafer evidence theory (D-S theory) [1,2] is widely used in decision making [3–6], information fusion [7–10] and uncertain information processing [11–16], etc., due to its advantages in handling uncertain information [17–22]. However, in D-S theory, the results with Dempster's combination rule are counterintuitive [23] when the given evidence highly conflicts with each other. Until now, how to manage conflict is an open issue in D-S theory. In recent years, hundreds of methods have been proposed to address this issue [8,24–37]. The methods how to manage conflict are generally divided into two categories:

(a) To modify the combination rule and redistribute the conflict. For example, Yager [38] proposed a re-assignment method of the mass to the empty set; Florea et al. [39] presented two combination rules named ACR (Adaptive Combination Rule) and PCR (Proportional Conflict Redistribution rule); Florea et al. [40] proposed a class of robust combination rules (RCR) which can automatically account for the reliability of information sources, etc.

(b) To modify the belief functions and remain Dempster' rule. Haenni [41] deems that the new rules can hardly keep the original properties that Dempster's rule have and promotes that we don't have to explore an alternative rule, and we could get the same effect by modifying the initial BPAs. Among these methods, the conflict management with conflict measurement is the most common way. Currently there occur a lot of methods of conflict quantification [42–48]. The classical conflict coefficient k is the mass of the combined belief assigned to the empty set. It focuses on the conflict among the focal elements but ignores the global consistency of the different pieces of evidence. Besides, k only considers the conflict information embedded in the non-intersection focal elements, but the differences of focal elements also lead to the conflict.

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To overcome this problem, researchers presented different solutions, such as Thomas and Nikhil [49], Jousselme et al. [50] and Martin et al. [51], they considered the conflict information from the non-intersection parts in their proposed conflict measurements. Some other researchers proposed some new factors to jointly measure the conflict with k. For example, Liu [52] used pignistic probability distance to form a two dimensional conflict model; Daniel [53] defined plausibility conflict to help to improve the ability of conflict measures; Lefevre and Elouedi [54] measured the conflict based on the distance of two BPAs and the mass of the empty set. Other methods, such as correlation coefficient, has been introduced to evidence theory to quantify the consistency of two BPAs [55–58]. For example, in [56], Song et al. defined a correlation coefficient by the cosine of angle with respect to two BPA vectors. In [55], the divergence degree between two BPAs is characterized by their proposed fuzzy nearness and correlation coefficient.

Although, the conflict measure methods are various and fruitful, but how to measure conflict efficiently is still inconclusive. The main reason is that the opinions about what is conflict and where it comes from are divergent. As in possibility theory [59] and the conflict measurement work in [42], in this paper, we measure conflict between information sources from the inconsistency of BPAs. More precisely, we take into consider both the non-intersection and the difference among the focal elements, and we propose a new correlation coefficient with several good properties which will be discussed in Section 4. Some numerical examples illustrate that the proposed correlation coefficient could effectively measure the conflict degree among belief functions.

The paper is organised as follows. In Section 2, the preliminaries D-S theory and the existing conflicting measurement are briefly introduced. Section 3 presents the new correlation coefficient and the proof of many desirable properties. In Section 4, some numerical examples and a real application are illustrated to show the efficiency of the proposed coefficient. Finally, a brief conclusion is made in Section 5.

2. Preliminaries

D-S theory was introduced by Dempster [1], then developed by Shafer [2]. Owing to its outstanding performance in uncertainty model and process, this theory is widely used in many fields [60–63].

Definition 2.1. Let $\Theta = \{\theta_1, \theta_2, \cdots \theta_i, \cdots, \theta_N\}$ be a finite nonempty set of mutually exclusive hypotheses, called a discernment frame. The power set of Θ, 2^Θ, is indicated as:

$$2^\Theta = \{\emptyset, \{\theta_1\}, \cdots \{\theta_N\}, \{\theta_1, \theta_2\}, \cdots, \{\theta_1, \theta_2, \cdots \theta_i\}, \cdots, \Theta\} \tag{1}$$

Definition 2.2. A mass function is a mapping **m** from 2^Θ to [0,1], formally noted by:

$$\mathbf{m} : 2^\Theta \rightarrow [0, 1] \tag{2}$$

which satisfies the following condition:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Theta} m(A) = 1 \tag{3}$$

When m(A)>0, A is called a focal element of the mass function.

In D-S theory, a mass function is also called a Basic Probability Assignment (BPA). Given a piece of evidence with a belief between $[0, 1]$, noted by $m(\cdot)$, is assigned to the subset of Θ. The value of 0 means no belief in a hypothesis, while the value of 1 means a total belief. And a value between $[0, 1]$ indicates partial belief.

Definition 2.3. Evidence combination in D-S theory is noted as $\oplus$. Assume that there are two BPAs indicated by m_1 and m_2, the evidence combination of the two BPAs with Dempster's combination rule [1] is formulated as follows:

$$m(A) = \begin{cases} 0, & A = \emptyset \\ \frac{1}{1-k} \sum\limits_{B \cap C = A} m_1(B) m_2(C) & A \neq \emptyset \end{cases} \tag{4}$$

with

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{5}$$

Where k is a normalization constant, called conflict coefficient because it measures the degree of conflict between m_1 and m_2.

$k = 0$ corresponds to the absence of conflict between m_1 and m_2, whereas $k = 1$ implies complete contradiction between m_1 and m_2. Note that the Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $k < 1$.