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The space and time behaviour of the constructal plate generating heat

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ABSTRACT

In this paper, a study of the elemental construct of an I-shaped fin cooling a heat generating plate is conducted in the framework of the constructal design method. Using the minimization of the maximum excess of temperature as an objective and considering that heat is generated in the conductive strip within a certain surface and with a different amplitude from the plate, the constructal design method permits to obtain a new optimal shape ratio (more general than the optimums found in the literature considering no-generation in the conductive strip or heat generated with the same amplitude in the two materials). Then, the thermal model used until here in stationary state (hypotheses on heat diffusion) is extended to take into account the time. The analytical solution obtained (verified numerically) is compared to classical heat diffusion through an engineering application (cooling an electronic device) where it keeps a good precision for times larger than 1% of the stationary time.

1. Introduction

Thermodynamics is a science studying the fluxes of mass, energy and entropy occurring in a system and its boundaries. Until recently geometrical dynamics in thermodynamics was not really taken into account to deal with optimization, and quantities such as exergy, entropy production minimization or finite thermodynamics were used [1–4].

Constructal theory appeared at first in the context of transportation sciences (focused on street network theory [5]) and thermal sciences (more precisely in the cooling of a heat generating area [6]). Several concepts such as irreversible process framework, equipartition of dissipation or entropy production have been used with this theory [7–10] and analytical solutions have been obtained [11–14]. Since the evidence of the efficiency of this theory, many processes have been analysed from engineering to nature [15–21].

The constructal theory is a physics principle which takes into account the importance of geometric orientation of fluxes and their configuration in the phenomenon under study. This physical principle is based on the fact that the configuration evolves to offer greater access to what flows in the system [19,22].

In the wake of constructal theory, constructal design is a method which permits to obtain the best geometrical configuration of flow currents in the system. Indeed, using particular objective and constraints allows to determine the optimal architecture of flows. The constructal design constructs shapes from an elemental object until larger objects (opposite of the fractal construction), and thus studies the different phenomena on multiple scales. The shapes obtained can be bounded to particular geometries like parabolic fractality [23–25]. Based on constructal design, several works permitted to obtain optimal configurations to enhance thermal performance of systems under convection [26–30] and conduction [31–36].

Recently, on the heat generating area problem, several possibilities have been studied concentrating on the variation of morphologies of the conductive strip (+, H, X, Y, V shapes) [37–42] with thermal contact resistance (I,T shapes) [43,44], considering size effect on conductivity of the conductive strip and studying nonuniform heat generation in the area [45–49].

In this paper, we extend the work of [6] focusing on the analytical solution related to the constructal design of the elemental construct (using hypothesis on heat diffusion in the two materials). We explore a heat generating plate through the constructal design method and take into account the influence of the area and amplitude of heat generation (in the conductive strip) on the optimums of the aspect ratio (where the works of [6] and [50] appear as particular cases). Then, we continue the study and develop an analytical solution of the elemental construct in time (verified numerically) which permits to obtain the thermal behaviour of the elemental construct and the time range of validity of the analytical solution compared to classical heat diffusion.

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2. Constructal study

2.1. Constructal origin in thermal science

In thermal sciences, the original constructal reasoning focuses on a heat generating slab area with conductivity (k_0) and dimensions ($H_0 L_0$) producing a constant heat power per unit volume ($\dot{r}$) which is drained by a conductive strip with conductivity (k_{CS}) and dimensions ($D_0 L_0$) producing also ($\dot{r}$) heat power per unit volume (non production of heat in the high conductive strip and evolution of drain topology have been also studied [50]). The constructal design method in this case uses the minimization of the dimensionless maximum excess of temperature in the plate as the objective submitted to the constraint of a finite amount of high conductive material.

Several hypotheses are usually used to model the heat generating plate. The first two are on the temperature field, considering that heat is conducted following the y direction in the slab and the x direction in the strip (where the slenderness hypothesis is used [51]):

$$\underbrace{\frac{d^2T}{dy^2} + \frac{\dot{r}}{k_0} = 0}_{\text{Generating slab}} \qquad \underbrace{D_0 \frac{d^2T}{dx^2} + H_0 \frac{\dot{r}}{k_{CS}} = 0}_{\text{Conductive strip}} \tag{1}$$

The hypotheses on the boundaries are taken such as:

$$\left.\begin{array}{l} \frac{dT}{dy} = 0 \quad \forall\, x \in [0; L_0] \text{ at } y = \pm H_0/2 \\ \frac{dT}{dx} = 0 \quad \forall\, y \in [-H_0/2; H_0/2] \text{ at } x = L_0 \\ \frac{dT}{dx} = 0 \quad \forall\, y \in \pm[D_0/2; H_0/2] \text{ at } x = 0 \end{array}\right\} \text{Adiabatic} \tag{2}$$

$$T(x, y) = T_a \quad \forall\, y \in \pm[0; D_0/2] \text{ at } x = 0\} \text{ Sink Temperature} \tag{3}$$

The temperature derived is then obtained [6]:

$$T(x, y) = \frac{\dot{r}}{2k_0}(H_0 y - y^2) + \frac{\dot{r} H_0}{k_{CS} D_0}\left(L_0 x - \frac{x^2}{2}\right) + T_a \tag{4}$$

The objective is then taken to minimize the dimensionless maximum excess of temperature in the area leading to the shape ratio:

$$\left(\frac{H_0}{L_0}\right)_{opt} = 2(\hat{k}\phi_0)^{-1/2} \qquad \hat{k} = \frac{k_{CS}}{k_0} \quad \phi_0 = \frac{d_0}{h_0} \tag{5}$$

Further optimization is then built replacing elemental volumes with n*th* construct of conductive strip minimizing the dimensionless maximum excess of temperature at each rank.

2.2. Constructal study on the semi generating plate

Using the fact that a horizontal symmetry exists on the elemental surface used in the literature (see Ref. [6] for example), we use a simpler geometry (Fig. 1) where $l_0 = L_0$, $d_0 = D_0/2$, $h_0 = H_0/2$.

We can now obtain the equations from constructal theory using a local energy balance without any accumulation of energy (stationary state):

- For the heat generating slab area, the energetic equilibrium is written assuming conduction flows only in the y-direction:

$$\varphi_{y+dy} dC = \varphi_y dC + \dot{r} dV \qquad dC = dx \cdot dz \qquad dV = dx \cdot dy \cdot dz \tag{6}$$

Where dz is taken as unity since we use a two dimensional hypothesis. Applying the Fourier law gives:

$$\frac{\varphi_{y+dy} - \varphi_y}{dy} = \dot{r} \qquad \varphi_y = -k_0\left(\frac{T_y - T_{y-dy}}{dy}\right) \tag{7}$$

The equation becomes finally:

$$-k_0\left(\frac{T_{y+dy} - 2T_y + T_{y-dy}}{dy^2}\right) - \dot{r} = k_0 \frac{\partial^2 T}{\partial y^2} + \dot{r} = 0 \tag{8}$$

For the conductive strip area, the energetic equilibrium is written assuming conduction flows only in the x-direction and moreover, since the limit conditions in the generating slab area are adiabatic, heat flows from the generating slab area to the conductive strip besides the classical conduction. Using the previous equation, we get the heat flowing from the heat generating area to the conductive strip:

$$\varphi_{0\to CS} = -\left(k_0 \frac{\partial T}{\partial y}\right) \cdot l_0 = -k_0 \int_{d_0}^{h_0} \int_0^{l_0} \left(\frac{\partial^2 T}{\partial y^2}\right) dx dy = \dot{r} l_0 (h_0 - d_0) \tag{9}$$

The energy flowing from the conductive strip area to the heat sink is then (assuming heat flows only in the x-direction):

$$\int_0^{d_0} \int_0^{l_0} \left(k_{CS} \frac{\partial^2 T}{\partial x^2} + \dot{r}\right) dx dy + \dot{r} l_0 (h_0 - d_0) \tag{10}$$

Integrating in the y-direction leads to:

$$\int_0^{l_0} \left(d_0 k_{CS} \frac{\partial^2 T}{\partial x^2} + d_0 \dot{r}\right) dx + \int_0^{l_0} \dot{r}(h_0 - d_0) dx \tag{11}$$

Finally the local form is obtained for the conductive strip area:

$$d_0 k_{CS} \frac{\partial^2 T}{\partial x^2} + \dot{r} h_0 \Rightarrow k_{CS} \frac{\partial^2 T}{\partial x^2} + \dot{r} \frac{h_0}{d_0} = 0 \tag{12}$$

The temperature distribution is then simply obtained:

1. When $\dot{r}$ is generated in ($h_0 \cdot l_0$):

$$T(x, y) = \frac{\dot{r}}{k_0}\left(h_0 y - \frac{y^2}{2}\right) + \frac{\dot{r} h_0}{k_{CS} d_0}\left(l_0 x - \frac{x^2}{2}\right) + T_a \tag{13}$$

Which leads to the optimum of the shape ratio (minimizing the dimensionless maximum excess of temperature):

$$\left(\frac{h_0}{l_0}\right)_{opt} = \sqrt{\frac{1}{\hat{k}\phi_0}} \tag{14}$$

2. When $\dot{r}$ is generated in $[(h_0 - d_0) \cdot l_0]$ (the conductive strip does not generate heat), the equation becomes:

$$T(x, y) = \frac{\dot{r}}{k_0}\left(h_0 y - \frac{y^2}{2}\right) + \frac{\dot{r}(h_0 - d_0)}{k_{CS} d_0}\left(l_0 x - \frac{x^2}{2}\right) + T_a \tag{15}$$

Which leads to the optimum of the shape ratio (minimizing the dimensionless maximum excess of temperature):

$$\left(\frac{h_0}{l_0}\right)_{opt} = \sqrt{\frac{(1 - \phi_0)}{\hat{k}\phi_0}} \tag{16}$$

We can infer that using the symmetry on height, $h_0 \to \pm H_0/2$ leads to a multiplication by the constant 2 on the shape ratio optimums whereas using the symmetry on length, $l_0 \to \pm l_0/2$ will lead to a multiplication by 1/2. Finally, if the two symmetries are used, the two coefficients vanish leading to the results presented previously.

It is necessary to outline that the boundary condition for the y direction diffusion used here is not correct (indeed the integration is taken between $y = 0$ and $y = h_0$) thus it has to be slightly corrected

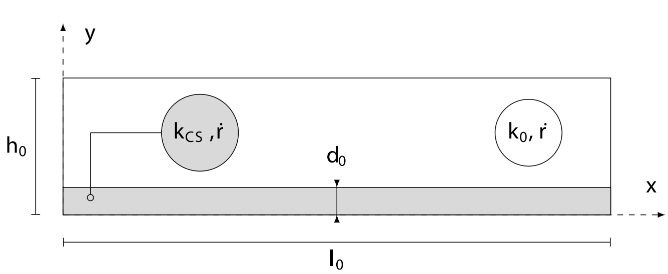

Fig. 1. Elemental surface used.