

## Partially-coherent spirally-polarized gradual-edge imaging

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### ABSTRACT

In this work we develop a partially-coherent spirally-polarized imaging system for generating gradual edge detection images. A rotating diffuser is used for producing illumination sources with tunable degree of coherence. Using a 4f configuration, the frequency content of the image is modified by means of a vortex half-wave retarder as a spatial filter. Accordingly, we analytically describe a spirally-polarized imaging system illuminated with a Gauss–Schell source using paraxial coherence-polarization theory. Numerical simulations and experimental results demonstrate the validity of our approach. We use a referenceless spatial content descriptor to assess the quality of the recorded images as a function of the coherence state of the source.

### 1. Introduction

The theory that describes the statistical properties of electromagnetic fields allows for a precise characterization of coherent phenomena [1–3]. This framework was extended to jointly describe coherence and polarization using a unified vector formalism [4–6]. Moreover, a generalization to partially coherent focused beams was introduced in [7,8]. On the other hand, the development of partially coherent sources with tunable degree of coherence is a topic of remarkable interest when dealing with experimental setups [9–13]. Very recently, some authors have proposed spatial filtering systems for producing beams with a given degree of coherence [14,15].

Vortex half-wave retarders (VHR) are useful for producing light beams with radial, azimuthal or spiral polarization and also as a way of enhancing the edges of the image [16,17]. Similar results are also obtained by means of spiral phase plates, as in [18]. In the present work we demonstrate the feasibility of producing tunable gradual edge detection on the recorded image by means of a partially-coherent spirally-polarized 4f imaging system. Interestingly, we found that the partial edge enhancement effect results in an improvement of the spatial quality content of the processed images. To the best of our knowledge, this is the first time this approach is described.

The paper is organized as follows: in Section 2, we mathematically describe a 4f processing system with a VHR filter placed at the Fourier plane and illuminated with a Gauss–Schell model source. Then, we present an experiment that demonstrates the results predicted by our theoretical approach. In Section 4, we discuss the behavior of the

recorded images by means of the analysis of the image histograms and the use of the Blind/Referenceless Image Quality Evaluator (BRISQUE) [19]. The use of this descriptor is very appropriate because it does not require any perfect reference to assess the quality of the image. Finally, we present our conclusions.

### 2. Mathematical description of a partially coherent, radially polarized 4f imaging system

Fig. 1 depicts a 4f optical system and the variables used in the present analysis. The object  $g(x_0, y_0)$  is placed at the front focal plane of lens  $L_1$  and illuminated with a totally polarized beam  $u(x_0, y_0)\vec{E}_0$ . Beam  $\vec{E}_0$  is linearly polarized i.e.  $\vec{E}_0 = (\cos \beta, \sin \beta)$ . Coordinate systems  $(x_0, y_0)$ ,  $(x', y')$  and  $(x, y)$  refer to the input, Fourier and output planes, respectively. Polar coordinates  $(r', \varphi')$  (Fourier plane) and  $(r, \varphi)$  (output plane) are also used. In what follows,  $u(x_0, y_0)$  is assumed to be a stochastic process, being  $\Gamma(x_1, y_1, x_2, y_2) = \langle u^*(x_1, y_1)u(x_2, y_2) \rangle$  the mutual coherence function.

A VHR filter  $\hat{M}$  is set at the back focal plane of  $L_1$  (Fourier plane). The Jones matrix of such a device is described by

$$\hat{M} = \begin{pmatrix} \cos \varphi' & \sin \varphi' \\ -\sin \varphi' & \cos \varphi' \end{pmatrix}. \quad (1)$$

The optical system is described by using paraxial propagation theory [20] and accordingly, the electric field at the back focal plane of tube lens  $L_2$  (output plane) reads:

$$\vec{E}(x, y) \propto \iint g(\bar{x} - x, \bar{y} - y) u(\bar{x} - x, \bar{y} - y) \vec{E}_0 \hat{N}(\bar{x}, \bar{y}) d\bar{x} d\bar{y} \quad (2)$$

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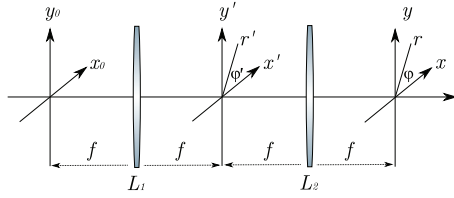


Fig. 1. 4f setup.

where matrix  $\hat{N}$  is the optical Fourier transform of  $\hat{M}$ , i.e.:

$$\hat{N}(r, \varphi) = \int \hat{M}(r', \varphi') \exp\left(i \frac{2\pi}{\lambda f} (r' r \cos(\varphi' - \varphi))\right) r' dr' d\varphi'. \quad (3)$$

Using the previous equation, it is straightforward to demonstrate that matrix  $\hat{N}$  reads

$$\hat{N}(r, \varphi) = 2\pi i \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \int_0^R J_1\left(\frac{2\pi r' r}{\lambda f}\right) r' dr', \quad (4)$$

where  $R$  is the radius of filter  $\hat{M}$  and  $J_1(x)$  is the Bessel function of the first kind and order 1. Interestingly, depending on the polarization direction of the input beam  $\vec{E}_0 = (\cos \beta, \sin \beta)$ , the electric field at the back focal plane  $\vec{E}(x, y)$  is spirally polarized. Then, the recorded intensity  $I(x, y)$  is described by

$$I(x, y) \propto \int g(x_1 - x, y_1 - y)^* g(x_2 - x, y_2 - y) \Gamma(x_1 - x, y_1 - y, x_2 - x, y_2 - y) \text{Tr} \left[ \hat{N}^\dagger(x_1, y_1) \vec{E}_0^\dagger \vec{E}_0 \hat{N}(x_2, y_2) \right] dx_1 dy_1 dx_2 dy_2. \quad (5)$$

Symbol  $\dagger$  and operator  $\text{Tr}[\ ]$  stand for conjugate transpose matrix and matrix trace respectively;  $x_1, y_1, x_2$  and  $y_2$  are dummy variables. In particular, for linearly polarized input beams  $\vec{E}_0 = (\cos \beta, \sin \beta)$  the trace term reads:

$$\text{Tr} \left[ \hat{N}^\dagger(x_1, y_1) \vec{E}_0^\dagger \vec{E}_0 \hat{N}(x_2, y_2) \right] = 4\pi^2 \cos(\varphi_1 - \varphi_2) \int_0^R J_1\left(\frac{2\pi r' r_1}{\lambda f}\right) r' dr' \times \int_0^R J_1\left(\frac{2\pi r' r_2}{\lambda f}\right) r' dr', \quad (6)$$

where  $\varphi_1$  and  $\varphi_2$  are the dummy angular coordinates related to  $x_1, y_1$  and  $x_2$  and  $y_2$ , respectively. For a partially coherent source that fulfills the Gauss–Schell model [2], the mutual coherence function  $\Gamma(\ )$  reads

$$\Gamma(x_1, y_1, x_2, y_2) = C \exp\left(-\frac{x_1^2 + x_2^2 + y_1^2 + y_2^2}{4\sigma^2}\right) \exp\left(-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2\mu^2}\right), \quad (7)$$

where  $C$  is a constant and  $\sigma$  and  $\mu$  describe the spatial width of the source and the longitude of coherence, respectively. In particular, special cases  $\mu/\sigma \rightarrow 0$  and  $\mu/\sigma \rightarrow \infty$  describe totally incoherent and totally coherent sources respectively. For an incoherent source, the mutual coherence function fulfills  $\Gamma(x_1, y_1, x_2, y_2) = \delta(x_1 - x_2)\delta(y_1 - y_2)$  and thus, the intensity  $I(x, y)$  becomes

$$I(x, y) = \int g(x_1 - x, y_1 - y)^* g(x_1 - x, y_1 - y) \text{Tr} \left[ \hat{N}^\dagger(x_1, y_1) \vec{E}_0^\dagger \vec{E}_0 \hat{N}(x_1, y_1) \right] dx_1 dy_1 = \int |g(x_1 - x, y_1 - y)|^2 \text{Tr} \left[ \hat{N}^\dagger(x_1, y_1) \vec{E}_0^\dagger \vec{E}_0 \hat{N}(x_1, y_1) \right] dx_1 dy_1. \quad (8)$$

Note that  $h_i(x, y) = \text{Tr} \left[ \hat{N}^\dagger(x, y) \vec{E}_0^\dagger \vec{E}_0 \hat{N}(x, y) \right]$  is the incoherent point spread function of the system.

For a full coherent source, the mutual coherence function is simply  $\Gamma(x_1, y_1, x_2, y_2) = u^*(x_1, y_1)u(x_2, y_2)$  and consequently, the intensity reads

$$I(x, y) \propto \int g^*(x_1 - x, y_1 - y) u^*(x_1, y_1) g(x_2 - x, y_2 - y) u(x_2, y_2) \text{Tr} \left[ \hat{N}^\dagger(x_1, y_1) \vec{E}_0^\dagger \vec{E}_0 \hat{N}(x_2, y_2) \right] dx_1 dy_1 dx_2 dy_2 = \left| \int g^*(\bar{x} - x, \bar{y} - y) u^*(\bar{x}, \bar{y}) \vec{E}_0 \hat{N}(\bar{x}, \bar{y}) d\bar{x} d\bar{y} \right|^2. \quad (9)$$

In order to provide more insight into the behavior of the described system, we calculated the intensity  $I(x, y)$  using Eq. (5) for different

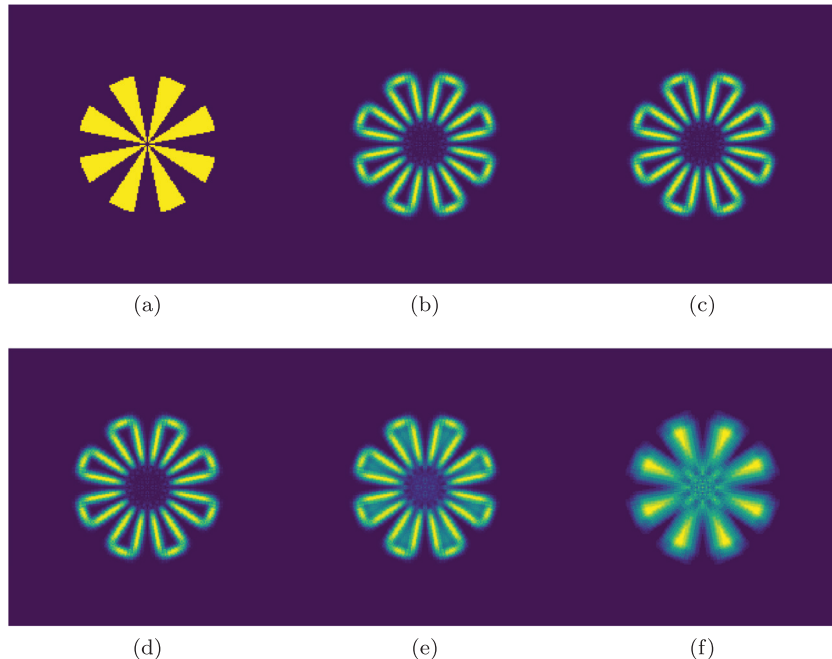


Fig. 2. Image formation simulations using partially coherent light. Images are presented in false color using the viridis colormap: (a) test object; (b)  $I(x, y)$  with  $\mu = 10^6$  mm; (c)  $I(x, y)$  with  $\mu = 100$  mm; (d)  $I(x, y)$  with  $\mu = 10$  mm; (e)  $I(x, y)$  with  $\mu = 3$  mm; (f)  $I(x, y)$  with  $\mu = 0.1$  mm.

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