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New optical solitary wave solutions of Fokas-Lenells equation in presence of perturbation terms by a novel approach

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ABSTRACT

A variety of new optical waves solutions of the Fokas-Lenells equation in presence of perturbation terms is investigated. A new approach is used, namely the generalized exponential function method. The physical meaning of the geometrical structures for some of these solutions is discussed for different choices of the free parameters present in the solutions. It is shown that the proposed methodology provides powerful mathematical tools for obtaining the exact traveling wave solutions of different nonlinear evolution equations.

1. Introduction

In the last years, the investigation of the complex waves propagation described by a certain type of nonlinear evolution equations (NLEE) has drawn the attention of the research community. The NLEE describing the composition and dynamical behavior of these waves in plasma physics, optical communications, laser technology, signal processing, and others, represent a significant challenge [1–5]. Some models with higher order and power-law nonlinearity are implemented to depict the optical solitons propagations in optical fibers [6,7]. The most appropriate way to comprehend the dynamics of these models is to find their exact solutions. The explicit solutions of these equations, if available, facilitate the verification of numerical researchers and aid in studying the stability analysis.

Different approaches are used in literature for calculating the exact solutions for the NLEE. Among these methods; the improved fractional sub-equation method [8,9], Kudryashov method and its extended form [10–13], the unified method and its generalized scheme [14–19], the homotopy perturbation method [20,21], and the new extended trial equation method [22,23].

In this paper, we study the Fokas-Lenells equation (FLE) in presence of perturbation terms [24,25] using the generalized exponential rational function (GER) method [26], that is a novel extended approach of the exponential rational function method [27,28].

The FLE is given by:

$$i\psi_t + a_1\psi_{xx} + a_2\psi_{xt}|\psi|^2(b\psi + i\sigma\psi_x) = \{\alpha\psi_x + \lambda(|\psi|^2\psi)_x + \mu(|\psi|^2)_x\psi\}. \tag{1}$$

In (1), $\psi(x, t)$ represents a complex field envelope, and x and t are spatial and temporal variables, respectively. Here, the first term represents the linear evolution of the pulses in nonlinear optical fibers, while the coefficient a_1 is the spatiotemporal dispersion (STD) and a_2 is the group velocity dispersion (GVD). Then the fourth term introduces the cubic nonlinear term, while the fifth term accounts for dispersion. On the righthand side of (1), the coefficient of α is the inter-modal dispersion (IMD), while λ is the self-steepening perturbation term and finally μ is the nonlinear dispersion (ND) coefficient.

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Based on these ideas this paper is organized as follows. The main aspects of the GER method are presented in Section 2. The discussion and application of this method to the FLE is discussed in Section 3. The physical meaning for some of the obtained solutions is presented in Section 4. Finally, the main conclusions outline are in Section 5.

2. Description of the method

In this section, we present the main steps used with the GER method for finding traveling wave solutions of NLEE. We suppose that a given NLEE for $u(x, t)$ to be in the form

$$\mathcal{N}(u, u_x, u_t, u_{xx}, \dots) = 0. \tag{2}$$

Using the transformations $u = u(\xi)$ and $\xi = x - \nu t$, we reduce Eq. (2) to the following nonlinear ordinary differential equation (NODE)

$$\mathcal{N}(u, u', u'', \dots) = 0, \tag{3}$$

where ν is a constant and $u' = \frac{du}{d\xi}$.

Step 1. The key of this method is to suppose that Eq. (3) has the formal solution

$$u(\xi) = A_0 + \sum_{k=1}^{N} A_k\, \Phi(\xi)^k + \sum_{k=1}^{N} B_k\, \Phi(\xi)^{-k}, \tag{4}$$

where

$$\Phi(\xi) = \frac{p_1\, e^{q_1 \xi} + p_2\, e^{q_2 \xi}}{p_3\, e^{q_3 \xi} + p_4\, e^{q_4 \xi}}. \tag{5}$$

The unknown coefficients A_0, A_k and $B_k\, (1 \le k \le N)$ and p_j and $q_j\, (1 \le j \le 4)$ are arbitrary real (or complex) constants to be determined, such that the solution (4) satisfies the NODE (3).

Step 2. After substituting (4) into Eq. (3) and collecting all terms, the left-hand side of (3) is converted into a polynomial $P(Y_1, Y_2, Y_3, Y_4)$ in terms of $Y_j = e^{q_j \xi}$ for $j = 1, \dots, 4$. If we set each coefficient of P to zero, then we derive a set of algebraic equations for p_j and $q_j\, (1 \le j \le 4)$, and for λ, ν, A_0, A_k, and $B_k\, (1 \le k \le N)$. This can be tackled with the aid of symbolic computation, such as Maple or Mathematica.

Step 3. After solving the algebraic equations in step 2, and substituting non-trivial solutions in (4), one can obtain the soliton solutions of Eq. (2).

3. Application of the GER method

To solve Eq. (1), first we need to apply the traveling wave transformation

$$\psi(x, t) = u(\xi)\, e^{i\eta(x,t)}, \quad \xi = x - \nu\, t, \tag{6}$$

where

$$\eta(x, t) = -\kappa x + \omega t + \theta. \tag{7}$$

Here, ν is the velocity of the soliton, κ is the frequency while ω is the soliton wave number and θ is the phase constant to be determined. Applying Eq. (6) into Eq. (1) we obtain the following pair of equations of real and imaginary components, respectively as

$$a_1 - a_2\, \nu\, u'' - (\alpha + \omega + a_1\kappa^2 - a_2\kappa\omega)u + (b - \kappa\lambda + \kappa\sigma)u^3 = 0, \tag{8}$$

and

$$(\nu + \alpha + 2a_1\kappa - a_2(\nu\, \kappa + \omega) + (3\lambda + 2\mu - \sigma)u^2)u' = 0. \tag{9}$$

From (9), the velocity of the soliton is obtained as

$$\nu = \frac{\alpha + 2a_1\kappa - a_2\omega}{a_2\kappa - 1},$$

provided $a_2\kappa \ne 1$. We also obtain the constraint condition of

$$3\lambda + 2\mu - \sigma = 0.$$

Balancing the terms of u^3 and u'' in Eq. (8) gives $3N = N + 2$, so $N = 1$. Hence, from Eq. (4), we assume the solution of Eq.(1) as:

$$u(\xi) = A_0 + A_1\Phi(\xi) + \frac{B_1}{\Phi(\xi)}. \tag{10}$$

where $\Phi(\xi)$ is giving by (5).