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Angular momentum of radiation at axial channeling

Vladimir Epp[a,*], Julia Janz[b], Margarita Zotova[c]

[a] Tomsk State Pedagogical University, ul. Kievskaya 60, 634061 Tomsk, Russia
[b] Tomsk Polytechnic University, pr. Lenina 30, 634050 Tomsk, Russia
[c] Russian Metrological Institute of Technical Physics and Radio Engineering, Mendeleevo, Moscow Region, Russia

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ABSTRACT

We study theoretically the angular momentum of radiation emitted at axial channeling of the fast particles in a crystal or in a bunch of micro- or nanotubes placed in strong magnetic field. It is shown that high energy particles channeled in the presence of magnetic field are effective source of vortex radiation in the X-ray and gamma-range of photons energies. We show that there are two factors that increase the angular momentum of emitted radiation in the presence of magnetic field. First, the magnetic field favours additional "twisting" of the channeled particles in a certain direction and, secondly, the particles that have the corresponding initial angular momentum are predominantly captured into the channeling mode. Dependence of the angular momentum of radiation on the value of magnetic field and on the incident angle is studied.

1. Introduction

Recently, the angular momentum of electromagnetic radiation has been widely explored theoretically and experimentally. The first theoretical and experimental research of such radiation, usually referred to as twisted light or vortex radiation, were devoted to laser radiation modified by astigmatic optical system or numerically computed holograms [1,2]. Further extensive bibliography on the history in this field one can find in the recent review [3].

Radiation carrying angular momentum can be also emitted by the vortex beams of charged particles as pointed out by Bliokh et al. [4]. Radiation in the X-ray range carrying the angular momentum was obtained by converting an X-ray beam [5] and by use of a helical undulator [6]. Various schemes of twisted photon beam production in undulators [7,8] and free electron lasers (FELs) [9–11] have been proposed. Cherenkov radiation and transition radiation caused by vortex electrons was studied theoretically by I. Ivanov and D. Karlovets [12,13]. The vortex radiation has found numerous applications both in classical and quantum optics condensed matter, high energy physics, optics, etc. (see the review [14] and references therein).

Radiation of high energy charged particles channeled in a solid crystal has certain advantages over the undulator or FEL radiation: i) it can have a much shorter wavelength, and ii) the radiation source is much more compact and mobile than the undulator or FEL. However, the main problem in generation of vortex radiation at axial channeling is that if a homogeneous beam of particles is incident on the entrance to the channel, then the initial angular momentum of the particles relative to the channel axis takes positive and negative values with equal probability. In order to have a predominant number of particles having trajectories of a certain helicity, we suggest placing the channel in a magnetic field aligned with the axis of the channel.

We consider channeling not only in a crystal but also in the micro- and nano-tubes. Conditions of the axial channeling in crystal are more favourable for negatively charged particles, though the positively charged particles can also participate in the axial channeling under certain conditions. The electric potential in micro- and nanotubes is formed by the walls of the tube, so the axial channeling is more effective for positively charged particles. The theory of channeling of positrons in carbon nanotubes is given by Klimov [15,16]. Channeling of protons and ions in carbon nanotubes is described in papers [17–19]. Channeling conditions in nanotubes are more favorable than in crystals due to a deeper potential well and a longer length of dechanneling [20]. Propagation of the channeled charged and neutral particle beams and radiation in channels of various nature inspires development of novel instruments and methods for applied research [21].

We show that there are two factors that increase the angular momentum of emitted radiation in the presence of magnetic field. First, the magnetic field favours additional "twisting" of the channeled particles in a certain direction and, secondly, the particles that have the corresponding initial angular momentum relative to the channel axis are

* Corresponding author.
E-mail addresses: epp@tspu.edu.ru (V. Epp), yyg@tpu.ru (J. Janz), zotova@vniiftri.ru (M. Zotova).

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predominantly captured into the channeling mode.

We assume that the electric potential in the channel is axially symmetric and quadratically dependent on the distance to the channel axis. This allowed us to solve the equations of motion and to find the critical channeling conditions. For the particles trapped in the channeling mode, we found the average angular momentum of radiation relative to the channel axis. In the next sections we discuss channeling in a single crystal, though all the results are applicable to radiation in any channel whose electric potential can be approximated by a quadratic function.

2. The particle trajectory

Suppose a crystal or a bunch of nanotubes is placed in the uniform magnetic field so that the tube axes or one of the crystal axes is parallel to the magnetic field. Let us find the particle trajectory. In order to obtain an analytical solution to equations of motion, we assume that the averaged potential of the channel is defined by [22,23]

$$\phi(r) = U_0\frac{r^2}{d^2}, \tag{1}$$

where d is "radius" of the channel, U_0 is the depth of the potential well, and r is the distance from the channel axis. We assume that the angle between the particle velocity and the channel axis is much smaller than the reciprocal of the relativistic factor $\gamma^{-1} = (1-\beta^2)^{1/2}$, where $\boldsymbol{v} = c\boldsymbol{\beta}$ is the particle velocity.

$$\sqrt{v_x^2 + v_y^2} \ll \gamma^{-1}v_z. \tag{2}$$

This implies that the particle energy can be considered constant, and the motion can be described by nonrelativistic equations for a particle with the mass $m = m_0\gamma$, where m_0 is the particle mass

$$\begin{aligned} m\ddot{x} &= -\tfrac{2eU_0x}{d^2} - \tfrac{e}{c}H\dot{y},\\ m\ddot{y} &= -\tfrac{2eU_0x}{d^2} + \tfrac{e}{c}H\dot{x},\\ m\ddot{z} &= 0. \end{aligned} \tag{3}$$

It follows from the last equation that the particle moves with a constant velocity v_z in the direction of the z-axis. Let us impose

$$\omega_1^2 = \frac{2eU_0}{md^2}, \quad \omega_2 = \frac{eH}{mc}.$$

The sign of ω_2 depends on the sign of H. The particle motion in the xy-plane is given by equations

$$\ddot{x} + \omega_1^2x + \omega_2\dot{y} = 0, \tag{4}$$

$$\ddot{y} + \omega_1^2y - \omega_2\dot{x} = 0. \tag{5}$$

This system can be solved in terms of

$$x(t) = a_1\cos\Omega_1t - a_2\sin\Omega_1t + b_1\cos\Omega_2t + b_2\sin\Omega_2t, \tag{6}$$

$$y(t) = a_1\sin\Omega_1t + a_2\cos\Omega_1t - b_1\sin\Omega_2t + b_2\cos\Omega_2t, \tag{7}$$

where the constants a_i and b_i are determined by the initial conditions, and the oscillation frequencies are equal to

$$\Omega_1 = \frac{1}{2}(\sqrt{4\omega_1^2 + \omega_2^2} + \omega_2), \tag{8}$$

$$\Omega_2 = \frac{1}{2}(\sqrt{4\omega_1^2 + \omega_2^2} - \omega_2). \tag{9}$$

The change of the magnetic field direction to the opposite leads to replacement of Ω_1 by Ω_2 and vice versa. In the absence of magnetic field $\Omega_1 = \Omega_2 = \omega_1$. In case of weak magnetic field ($\omega_2 \ll \omega_1$) the particle motion consists of harmonic oscillations with two slightly different frequencies $\Omega_{1,2} = \omega_1 \pm \Delta\omega$, where

$$\Delta\omega = \frac{eH}{2mc}.$$

This frequency shift is equal to the frequency shift due to the Zeeman effect and has the same nature.

We denote by x_0, y_0 the initial coordinates of the particle in the plane xy, and by v_{0x}, v_{0y} the components of its initial velocity in this plane. Then

$$\begin{aligned} a_1 &= \tfrac{1}{\omega}(v_{0y} + x_0\Omega_2), \quad a_2 = \tfrac{1}{\omega}(-v_{0x} + y_0\Omega_2),\\ b_1 &= \tfrac{1}{\omega}(-v_{0y} + x_0\Omega_1) \quad b_2 = \tfrac{1}{\omega}(v_{0x} + y_0\Omega_1), \end{aligned} \tag{10}$$

where $\omega = \Omega_1 + \Omega_2$.

It follows from the law of motion (6) and (7) that the radius vector of a particle in the plane xy can be represented as the sum of two vectors $\boldsymbol{r}(t) = \boldsymbol{a}(t) + \boldsymbol{b}(t)$, where

$$\boldsymbol{a}(t) = (a_1\cos\Omega_1t - a_2\sin\Omega_1t,\ a_1\sin\Omega_1t + a_2\cos\Omega_1t), \tag{11}$$

$$\boldsymbol{b}(t) = (b_1\cos\Omega_2t + b_2\sin\Omega_2t,\ -b_1\sin\Omega_2t + b_2\cos\Omega_2t), \tag{12}$$

are vectors of constant length $a = \sqrt{a_1^2 + a_2^2}$ and $b = \sqrt{b_1^2 + b_2^2}$, rotating in the opposite directions with angular velocities Ω_1 and Ω_2 respectively. If the particle is incident on the channel at a small angle to its axis and $\omega_2 \ll \omega_1$, then the trajectory is a precessing ellipse as shown in Fig. 1.

3. Channeling condition

Let us determine the initial conditions which ensure that the particle will be captured into channeling. In the absence of magnetic field, a particle is captured into channeling if the incidence angle (angle between the velocity vector and the channel axis) is less than the critical Lindhard angle

$$\frac{v_0}{v_z} \leqslant \alpha_L = \frac{1}{\beta_z}\sqrt{\frac{2eU_0}{mc^2}}, \tag{13}$$

where $\beta_z = v_z/c$, $v_0 = (v_{0x}^2 + v_{0y}^2)^{1/2}$. The condition for the particle to be trapped in the channel in the presence of magnetic field can be found from the requirement that the maximum distance which a particle can move away from the channel axis without leaving the channeling mode is equal to the channel radius d. As the maximum offset of the particle from the axis is equal to $r_m = a + b$ the channeling condition takes the following form

$$\sqrt{a_1^2 + a_2^2} + \sqrt{b_1^2 + b_2^2} \leqslant d. \tag{14}$$

Substituting here the inequality sign by equality, we obtain the

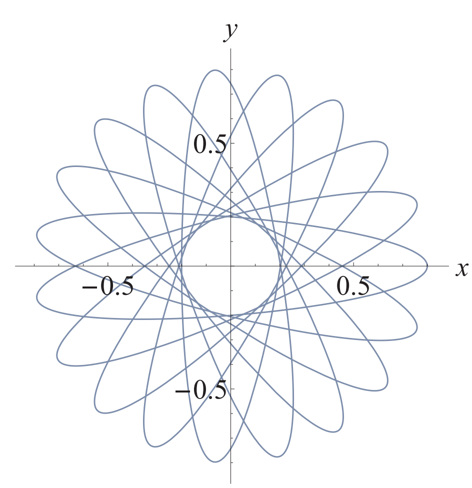

Fig. 1. An example of the particle trajectory at axial channeling in the presence of magnetic field.