High accuracy proton relative stopping power measurement

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ABSTRACT

Proton therapy is a fast growing treatment modality for cancer and is in selected cases preferred over conventional radiotherapy with photons because of the highly conformal dose distribution that can be achieved with protons due to their steep dose gradients. However, these steep gradients also make proton therapy sensitive to range uncertainties. Proton ranges are calculated from proton stopping powers relative to that in water (Relative Stopping Power, RSP). The RSPs needed for a treatment plan can be estimated from CT (Computed Tomography) data of a patient. High accuracy reference values of RSPs are required to assess the accuracy of these CT based estimates. In this paper we present a water phantom that enables accurate measurement of depth dose profiles in water. Experimental RSPs with a relative standard uncertainty smaller than 0.4% (1σ) for samples with a water equivalent thickness of about 2 cm can be derived from the measured depth dose distributions. Most CT based RSP estimates use an approximate RSP model based on the Bethe-Bloch formula without the shell, density, Barkas and Bloch correction. In the Geant4 Monte Carlo code these corrections are included and RSP calculations with this code are expected to be more accurate. In this work, a set of 32 well defined (composition and density), mostly clinically relevant materials is used to assess the correspondence between RSPs that were measured, that were estimated from the approximate RSP model and that were calculated from Monte Carlo simulations. With the measured RSPs we provide a ground-truth bench mark to test the validity of RSPs derived from CT imaging and Monte Carlo simulations.

1. Introduction

The potential advantage of protons over conventional radiation modalities (photons, electrons) for radiotherapy of cancer was first mentioned by Wilson in 1946 [1]. Protons have a finite range and local high dose region which facilitate a higher conformity to the tumor and less dose to the surrounding healthy tissues as compared to conventional irradiation with photons. Accurate positioning of the local high dose region is critical for exploiting the benefit of protons over photons. In proton therapy, each tissue is characterized by a proton stopping power relative to water (relative stopping power, RSP) that is derived from X-ray computed tomography (CT) data of the patient. Uncertainties in these RSPs introduce range uncertainties which have to be taken into account in treatment planning by using safety margins. These safety margins reduce the possibility to fully exploit the advantage of proton irradiations by limiting beam angles and increasing dose to healthy and sometimes critical tissues surrounding the tumor. Different methods have been proposed to derive RSPs from CT data of the patient. Single energy CT (SECT) methods typically correlate measured CT numbers to RSPs based on calculated CT numbers and RSPs for tissue substitutes or average tissue compositions [2,3]. Dual energy CT (DECT) provides measured CT numbers for two different spectral photon distributions and allows determination of the relative electron density and an effective atomic number [4–10]. RSPs can be calculated from the measured relative electron density and a relation between this effective atomic number and the mean excitation energy in the Bethe-Bloch formalism for calculation of stopping powers [10,11]. To assess the validity of the different CT-based methods for calculation of RSPs an accurate method for measurement of RSPs is needed.

In this paper we introduce an accurate method to derive RSPs from measured depth dose distributions using a water phantom in a proton beamline. Measured RSP's are presented for a set of 32 well defined (composition and density), mostly clinically relevant materials. These measured RSPs are used as ground-truth values to assess the accuracy of the Bethe-Bloch formula without the higher order corrections and of Geant4 Monte Carlo simulations to predict RSPs.

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2. Materials and methods

2.1. Accuracy of stopping power calculations based on the Bethe-Bloch equation

The total proton stopping power is due to energy transfer of the proton to the electrons (electronic stopping) and to the nuclei (nuclear stopping) of the target material. Electronic stopping causes ionization and excitation of target atoms and energy loss of the proton. Nuclear stopping changes the direction of the proton and the intensity of the incoming proton beam but contributes less than 0.1% to the total stopping power above 0.4 MeV [12] and is thus not relevant for range calculations. The Bethe-Bloch formula that describes the electronic stopping power of protons can be expressed as [13,14]

$$S = -\frac{dE}{dx} = \left(\frac{4\pi r_e^2 m_e c^2}{\beta^2}\right)\left(N_A\rho\frac{Z}{A}\right)L(\beta) \tag{1}$$

with r_e the classical electron radius and $m_e c^2$ the electron rest energy with c the speed of light in vacuum. The first factor in the energy loss is proportional to $1/\beta^2$ with $\beta = v/c$ and v the proton velocity. The second factor is the electron density of the target material ρ_e, which equals $N_A\rho Z/A$ with mass density ρ, Avogadro's number N_A, atomic number Z and atomic weight A. The last factor is the stopping number $L(\beta)$, which is the sum of the primary stopping number L_0, the Barkas correction L_1, the Bloch correction L_2 and higher order corrections which are negligible compared to L_1 and L_2.

$$L(\beta) = L_0(\beta) + L_1(\beta) + L_2(\beta) + \cdots \tag{2}$$

The primary stopping number L_0 can be expressed as

$$L_0(\beta) = f(\beta) - \ln\langle I\rangle - \frac{C}{Z} - \frac{\delta}{2} \tag{3}$$

where $f(\beta)$ is defined as

$$f(\beta) = \ln\left(\frac{2m_e c^2\beta^2}{1-\beta^2}\right) - \beta^2 \tag{4}$$

The mean excitation energy term $\ln\langle I\rangle$ takes into account the electronic structure of the target material. The mean excitation energy $\langle I\rangle$ is defined as the effective value (averaged over all possible electron states (ionization, vibration and excitation)) of the minimum energy transfer in a collision. The shell correction term C/Z addresses the fact that when the proton velocity decreases from relativistic energies the proton velocity is no longer much larger than the bound electron velocity as required for the Bethe-Bloch theory to be valid. The density effect term $\delta/2$ corrects for polarization effects in the target material, reducing the stopping power by a decrease of the assumed free-space electromagnetic field of the proton by the dielectric constant of the target material [14]. We have estimated from information in the literature the relative importance of the different correction terms with respect to the term $f(\beta) - \ln\langle I\rangle$.

Shell corrections (C/Z) become especially important for the inner shell electrons of the heavier elements. They have been calculated using hydrogenic wave functions [15] or the local density approximation [13]. Both methods seem to give consistent results. Low Z elements have the smallest correction. For elements most relevant for proton therapy $(Z < 20)$ the absolute value of the correction term is around 0.15 between 1 and 10 MeV and decreases to < 0.1 between 10 and 40 MeV and < 0.05 between 40 and 250 MeV. This corresponds to a relative decrease of the stopping power of 3–4% for energies between 1 and 10 MeV and approximately 1–1.5% between 10 and 40 MeV and $< 1\%$ for energies between 40 and 250 MeV.

The density effect $(\delta/2)$ only becomes relevant if the kinetic energy of the proton exceeds its rest energy and is therefore of limited importance for clinically used proton energies (up to 250 MeV). The absolute contribution of the density effect is estimated (from Fig. 11 in

[13]) to be smaller than 0.01 for all elements and energies below 200 MeV. Because $f(\beta) - \ln\langle I\rangle$ is larger than 5 for Z between 1 and 20 and energies larger than 10 MeV this implies a correction $< 0.2\%$ to the stopping power.

The Barkas correction (L_1) corrects for the higher density of target electrons in the vicinity of the positively charged proton. For low energy protons this effect becomes important because the target electrons have time to move towards the stopping protons. Ashley et al. [16] derived an empirical formula for low energy projectiles to approximate this effect. This formula was used by Bichsel [12] who reports a Barkas correction of 0.36% for 10 MeV protons on aluminium. Using the empirical approach of Ziegler (Eq. 35 in [13]) we estimate that the Barkas correction is smaller than 0.9% for energies above 10 MeV for Z between 1 and 20.

The Bloch correction (L_2) originates from close collisions of protons with target electrons and mostly depends on the proton energy and little on the target material. From Eq. (5) from Bichsel [12] we conclude that the Bloch correction is smaller than 0.2% for energies between 5 and 10 MeV and $< 0.1\%$ above 10 MeV.

Bragg and Kleeman [17] proposed an additivity rule for mass stopping powers S/ρ of elements to determine the mass stopping power of a mixture or compound. Following this Bragg additivity rule the mean excitation energy $\ln\langle I\rangle$ of a compound can be approximated by

$$\ln\langle I\rangle = \frac{\sum_k \omega_k \frac{Z_k}{A_k}\ln\langle I_k\rangle}{\sum_k \omega_k \frac{Z_k}{A_k}} \tag{5}$$

with the mass fraction ω_k and mean excitation energy $\langle I_k\rangle$ of element k in the compound. The validity domain and the accuracy of the Bragg additivity rule have not been well established. The Bragg additivity rule does not account for different states of aggregation and chemical binding between atoms in a molecule. The effect of the state of aggregation is for water (vapor, liquid or ice) the largest for proton energies of 50–100 keV [18]. From core and bond corrections applied in the software package SRIM [14] to account for chemical binding we conclude that the corrections may amount to 6–7% in the stopping region (up to 1 MeV) and are applied as a constant (energy independent) scaling factor. For energies above 3–4 MeV no corrections are applied in SRIM. The uncertainty in the $\langle I\rangle$ values of the elements is difficult to estimate but experiments suggest that tabulated elemental $\langle I\rangle$ values [19,20] are too low for the elements [21], leading to underestimation of $\langle I\rangle$ values for compounds when calculated with the Bragg additivity rule [22].

The range of a 10 MeV proton in water in the continuous slowing down approximation (CSDA) is 1.23 mm and this decreases rapidly for lower energies: for 5 MeV protons it has already decreased to 0.36 mm [23]. Consequently, for energies below 10 MeV, correction terms which contribute less than 10% (equivalent to 0.1 mm water) are not relevant for the total range prediction with the Bethe-Bloch formula. The overall contribution of the correction terms is dominated by the shell correction which amounts -1.5 to -1% for energies between 10 and 40 MeV and -1 to 0% between 40 and 250 MeV. The Barkas correction contributes $< 1\%$ above 10 MeV. These corrections partly cancel out due to their respectively negative and positive sign. As the higher energies contribute most to the range (40 MeV protons have a CSDA range of only 1.49 cm [23]), the total effect on the range of all correction terms is estimated to be less than 1–1.5% for protons with clinically relevant energies.

Consequently, the electronic stopping power of a material can be approximated within this estimated accuracy of 1–1.5% by

$$S = -\frac{dE}{dx} = \frac{4\pi r_e^2 m_e c^2}{\beta^2}\rho_e(f(\beta) - \ln\langle I\rangle) \tag{6}$$

provided the value used for $\langle I\rangle$ is correct.

The electronic stopping power relative to water (relative stopping power, RSP) can then be approximated by