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# Kac–Moody groups and cosheaves on Davis building

Katerina Hristova, Dmitriy Rumynin *,[1]

*Department of Mathematics, University of Warwick, Coventry, CV4 7AL, UK*

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ABSTRACT

We investigate smooth representations of complete Kac–Moody groups. We approach representation theory via geometry, in particular, the group action on the Davis realisation of its Bruhat–Tits building. Our results include an estimate on projective dimension, localisation theorem, unimodularity and homological duality.

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Our investigation of representation theory of Kac–Moody groups aims to combine two known lines of inquiry. Bernstein in his 1992 lectures in Harvard [1] proposed to look at representation theory of $p$-adic groups through a geometric prism, à la Klein. A $p$-adic

* Corresponding author.
*E-mail addresses:* K.Hristova@warwick.ac.uk (K. Hristova), D.Rumynin@warwick.ac.uk (D. Rumynin).

[1] Associated member of Laboratory of Algebraic Geometry, National Research University Higher School of Economics, Russia.

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group $H$ acts on a space, its Bruhat–Tits building $\mathcal{BT}$. A careful study of this action brings new, useful insights into representation theory of $H$. This approach culminated in the 1997 seminal work by Schneider and Stuhler [22] where they developed a systematic approach for passing from representations to equivariant objects on $\mathcal{BT}$, an ultrametric rendition of the Beilinson–Bernstein localisation.

The second line comes from the 2002 influential work by Dymara and Januszkiewicz. They pioneered a method for computing cohomology of a Kac–Moody group $G$ by studying cohomology of $\mathcal{BT}$ and its Davis realisation $\mathcal{D}$ [8].

In the present paper we examine the smooth representations of a Kac–Moody group $G$ by localising them over $\mathcal{D}$. In a certain sense, we unify the two lines of inquiry described above. A natural question is whether it is possible to use $\mathcal{BT}$ rather than $\mathcal{D}$. It is possible only for those Kac–Moody groups $G$ that are hyperbolic in the following sense: any proper Dynkin subdiagram is of finite type. In particular, affine Dynkin diagrams are hyperbolic, so our results are applicable to algebraic groups over local fields and their Bruhat–Tits buildings.

Let us now explain the content of the present paper. We strive to cover the correct generality in our results, the generality where our proofs work. The price we pay for this is that the different sections of the paper have different assumptions. Let us go section by section explaining our results and our assumptions.

In Section 1 we collect useful results about Haar measure on a locally compact totally disconnected topological group $G$. Most of this section is covered by Vigneras' book [24, I.1–I.2] but we find it essential to set up the notation and review some facts for the benefit of the reader. A criterion for unimodularity (Proposition 1.2) is new. Another accessible source is the book by Bushnell and Henniart [4], although they assume unimodularity and work in characteristic zero.

In Section 2 we keep the same assumptions on the group $G$ as in Section 1, in particular, $G$ is not necessarily unimodular. In perspective, we would like to cover the group $\mathrm{GL}_n(\mathbb{K})$ over a local field $\mathbb{K}$. When $\mathrm{GL}_n(\mathbb{K})$ acts on $\mathcal{BT}$, the stabilisers are not compact, just compact modulo centre. So we choose a central subgroup $A$ of $G$, modulo which we can effectively describe geometry and representation theory. In particular, we introduce the abelian category $\mathcal{M}_A(G)$ of $A$-semisimple smooth representations of $G$ over a field $\mathbb{F}$. We show that $\mathcal{M}_A(G)$ is equivalent to a category of representations of a Hecke algebra (Proposition 2.5). The pay-off is existence of enough projectives in $\mathcal{M}_A(G)$ (Corollary 2.7).

We study these projectives in Section 3 and contemplate projective resolutions. If $(P_\bullet, d_\bullet)$ is a resolution of the trivial module, then $(P_\bullet \otimes V, d_\bullet \otimes I_V)$ is a projective resolution of any module $V$ (Lemma 3.3). At this point we prove our first main theorem à la Bernstein (Theorem 3.5): if $G$ acts on a contractible simplicial set $\mathcal{X}_\bullet$, the projective dimension of $\mathcal{M}_A(G)$ is bounded above by the dimension of $\mathcal{X}_\bullet$. Interestingly enough, we could not find a discussion of group action on a simplicial set in the literature, so we feel compelled to include some deliberations on this topic.

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