Journal of Algebra 515 (2018) 360–388

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

On the graded dual numbers, arcs, and non-crossing partitions of the integers

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ARTICLE INFO

Article history:
Received 9 April 2018
Available online 7 September 2018
Communicated by Volodymyr Mazorchuk

Keywords:
Triangulated categories
Thick subcategories
Non-crossing partitions

ABSTRACT

We give a combinatorial model for the bounded derived category of graded modules over the dual numbers in terms of arcs on the integer line with a point at infinity. Using this model we describe the lattice of thick subcategories of the bounded derived category, and of the perfect complexes, in terms of non-crossing partitions. We also make some comments on the symmetries of these lattices, exceptional collections, and the analogous problem for the unbounded derived category.

Contents

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https://doi.org/10.1016/j.jalgebra.2018.08.023
0021-8693/

1. Introduction

There is, by now, a small industry of classifying well-behaved subcategories of triangulated categories. Here well-behaved usually means, at a minimum, closed under suspension, cones, and summands but frequently also entails being closed under some additional operation, such as a tensor product if the triangulated category is monoidal. This program of understanding the coarse structure of triangulated categories has its genesis in the pioneering work of Devinatz, Hopkins, and Smith [6], Neeman [14], and Benson, Carlson, and Rickard [4], and has led to a great deal of progress in algebra, geometry, and topology.

The aim of this article is to contribute another piece to the puzzle and advertise a direction that is in need of further investigation. In some sense, following the blueprint provided by the classification theorems mentioned above and culminating in the work of Balmer [3], we now have a very good understanding of thick tensor ideals in essentially small tensor triangulated categories. One gets a 'continuous' classification in terms of nice subsets of an associated topological space; all three of the works mentioned above fit into this paradigm.

On the other hand, there are many triangulated categories, arising for instance from representations of finite dimensional algebras, which do not necessarily naturally carry a reasonable monoidal structure. Our understanding of these cases is still extremely limited and there are few general techniques for describing the associated lattices of thick subcategories. However, certain examples are, at least partially, understood. Ingalls and Thomas [10] computed the lattice of thick subcategories for the bounded derived categories of simply laced Dynkin quivers (as well as in affine type). Here, the classification is purely combinatorial – there is no 'continuous part' – and thick subcategories correspond to non-crossing partitions of the relevant Dynkin type.

More generally, work of Igusa, Schiffler, and Thomas [9] (cf. also Krause's paper [13, Theorem 6.10]) gives insight into the combinatorial aspect of the classification problem more generally. They give a classification of thick subcategories coming from exceptional collections in bounded derived categories of finite acyclic quivers via a more general