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Level sets-based image segmentation approach using statistical shape priors

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ABSTRACT

A robust 3-D segmentation technique incorporated with the level sets concept and based on both shape and intensity constraints is introduced. A partial differential equation (PDE) is derived to describe the evolution of the level set contours. This PDE does not contain weighting parameters that need to be tuned, which overcomes the drawbacks of other PDE approaches. The shape information is collected from a set of co-aligned manually segmented contours of the training data. A promising statistical approach is used to get the distribution of the intensity gray values. The introduced statistical approach is built by modeling the empirical PDF (normalized histogram of occurrences) for the intensity level distribution with a linear combination of Gaussians (LCG) incorporating both negative and positive components. An Expectation-Maximization (EM) algorithm is modified to deal with the LCGs, and we also proposed an EM-based sequential technique to acquire a close initial LCG approximation for the modified EM algorithm to start with. The PDF of the intensity levels is incorporated in the speed function of the moving level set to specify the evolution direction. Experimental results show how accurately the approach is in segmenting various types of 2-D and 3-D datasets comprising medical images.

1. Introduction

The level set method was inspired by Osher and Sethian [1] as a simple and versatile method for computing and analyzing the motion of an interface contour Γ in two or three dimensions. That interface bounds a connected or possibly a multiply connected region. The goal is to compute and analyze the subsequent motion of this evolving curve/surface under a velocity field. This velocity can depend on position, time, geometry of the interface (e.g., its normal or its mean curvature), and external physics. The interface is captured for later time as the zero level set of a smooth (at least Lipschitz continuous) function $\phi(\mathbf{x}, t)$. Such level set function ϕ is defined as positive inside the region, negative outside, and is zero on Γ. The original idea behind the level set method was a simple one. Given an interface contour Γ in 2-D/3-D, bounding a (perhaps multiply connected) open region, we wish to analyze and compute its subsequent motion under a velocity field.

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A deformable model is a curve in a 2-D digital image or a surface in a 3-D image that evolves to outline a desired object. The evolution is controlled by internal and external forces combined, together with user defined constraints, into internal and external energy terms, respectively. Introduced first by Kass et al. [2], the models gave rise to one of the most dynamic and successful research areas in edge detection, image segmentation, shape modeling, and visual tracking. By representation and implementation, deformable models are broadly categorized into parametric and geometric classes. In [3], the proposed stochastic force has been embedded in the geometric deformable models (level sets), and in this paper, the main focus is to discuss the geometric deformable models and how to deform it under a particular stochastic force till it captures the object properties of interest.

Performance of the deformable models depends on proper initialization, on efficiency of the energy minimization process, and on adequate selection of force functions and energy functionals. Specifically, the original iterative minimization in [2] based on a closed form solution of Eulerian differential equations that specify the desired minimum turns out to be unstable and usually gets trapped in local minima. Also, it is difficult to constrain the force functions or the whole energy function. Amini et al. [4] pointed to these shortcomings of the minimization process in [2] and improved it by representing a snake as a linked chain of control points and using discrete dynamic programming to minimize the total energy related to the chain. This approach allows for rigid constraints on the energy function and more minimization stability. However, its control parameters must be adjusted very carefully, and the process remains too time consuming. Another advanced greedy algorithm proposed by Williams and Shah [5] has linear time complexity both in the number of control points and in the number of their neighbors taken into account for energy minimization. It is much more stable and is simultaneously more than an order of magnitude and faster than previous techniques. Wong et al. [6] developed a more flexible segmented snake. Their model is able to handle regions with relatively sharp corners due to recursive split and merge procedure dividing a boundary into segments to approximate them locally. An alternative flexible snake model in [7] is based on B-spline representation and multiple stages energy minimization.

In spite of good segmentation results for objects of relatively simple shapes, the above conventional deformable models have serious drawbacks. Most of them are slow compared to other segmentation techniques, and the model evolution frequently stops well before approaching a complicated object boundary with concavities. Also, to initialize the model, typically a closed curve has to be interactively drawn near the desired boundary, and this manual step hinders its use in many applications [8].

Leventon et al. [9] used prior shape with deformable models in order to control the evolution of the active contours. They obtained their shape model by applying the principal component analysis (PCA) to the training shape signed distance maps. Shen and Davatzikos [10] used attribute vectors with the deformable models which identify the surrounding geometry of the model points during the deformation. Based on [9], Tsai et al. [11] presented a deformable model based segmentation technique in which the segmenting curve is represented implicitly using a parametric model. This eliminates the need of obtaining correspondence between points at the time of training and hence more robustness to variations in the segmenting curve. The implicit shape is parametrically represented using the mean and Eigen shapes resulting from the PCA of the signed distance maps of the training shapes. Later on, this method was extended by Tsai et al. [12] to handle the simultaneous segmentation of multiple objects by using multiple signed distance maps to implicitly represent the shapes. PCA is applied to these distance maps and co-variation between various shapes is acquired. This time, the cost function involved in the segmentation is based on mutual information. However inconsistent modeling can occur due to the non-closure of distance functions with linear operation. This was handled by Pohl et al. [13] where they embedded the signed distance maps into the logarithm of the odds linear space. Also for segmentation based on level sets, Yang and Duncan [14] performed 3-D segmentation using joint prior shape and intensity model making use of the dependency between shape and intensity of objects, where they developed a maximum a posteriori (MAP) model using the joint information. Huang et al. [15] presented a problem of energy minimization that combines both segmentation and registration. A level set of a distance function of a higher dimensionality embeds either the evolving surface or curve which is iteratively registered using the shape model. Rousson et al. [16] employed distance functions to build a shape model using shapes in the training dataset in order to be used for a 3-D level set shape based segmentation technique.

This paper proposes a novel and promising level set based technique for segmentation and is dependent on the information from both the shape and the intensity. The information about the former is extracted from a training set of co-aligned shapes. The value of the gray level intensity is used to form the probability density functions (PDFs) of both the object and the background. The mentioned functions are found using a generalized version of our novel approach which is named the modified Expectation Maximization (EM) [17]. The modified EM uses a linear combination of Gaussians (LCG) that have negative and positive components to estimate the density function. Using the estimated PDFs in a variational manner lead to accurate and fast segmentation. The main contributions of this paper over our previous work are extending the algorithm to work in both 2-D and 3-D, the integration of LCG intensity model with the shape prior, and also, introducing a new speed function formula that is able to take both the intensity and shape information to guide the propagation of the level set.

2. Evolutionary curve/surface based on level set model

Shape delineation is the primary task in shape analysis. Such representation is essential in the computer vision field and several medical imaging applications such as registration and segmentation. There are several shape representation techniques explained in [18–20]. Despite the fact that some of those methods are powerful sufficiently to acquire local