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Non-fragile state estimation for delayed fractional-order memristive neural networks☆

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ABSTRACT

The issue of non-fragile estimation for fractional-order memristive system is provided in this paper. By endowing the Lyapunov technique, the corresponding works that ensuring the globally asymptotic stability of the error model are presented, which can be calculated efficiently. In the end, the analytical methods are voiced by two simulations.

1. Introduction

Considering the mathematical relations among current, voltage, charge and magnetic flux, the existence of memristor was envisioned out by Prof. Chua in 1971 [1], and this device has been theoretically suggested by D. Strukov, et al. in 2008 [2]. Since, this brand new circuit element has caused the world's attention from the research organization considering its future applications, especially in the next generation brain-like 'neural' computers.

Memristor is related to the charge q and magnetic flux φ that across the device, and this two physical quantity also determine the resistance of memristor. Moreover, as one knows, the $v-i$ plane of memristor is a hysteretic curve, i.e., when any bipolar periodic signal flows through this device, its resistance schematic exhibits a hysteresis phenomenon. This special curve is generally considered as a fingerprint of memristor. Besides, a memory function signifies another advantage of this devices, i.e., once a computer is build by memristor, it is able to storage the information appeared in the previous time. More importantly, even when the power is failure, the information will be arisen as long as it is connected again.

In the artificial neural networks, a memristive model can be reached once the resistor was replaced by memristor, subsequently, lots of interesting works have been appeared, among which, the information capacity may be the most compelling advantage [3]. In particular, this noticeable merit makes the memristive model contains larger information storage capacity. Thus, this kinds of system possess much more computation power, and it is high time to consider the dynamic behaviors of neural networks especially for the memristive one [4–10].

Considering the complexity and the supermassive dimension of a memristive model, it is very hard to obtain the accurate information of the system. Besides, the unreliable measurements, packet dropouts together with the missing measurements might lead that the states information could not fully reached and only relies on the network outputs. Thus, it is necessary

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to estimate the information via measurements. To answer these problems, the state observation topic speak loudly during the past few years and some remarkable works are reported in [7], [11].

It is worth stressing that most of the existing works only tackle with the uncertainties appeared in the system, while the uncertainties arisen in the observers have not been well addressed, which raised a ‘fragility’ topic [12–15]. The essence of this problem is to determine the level of computer accuracy, as a result, the system specifications can be guaranteed.

Fractional calculus mainly cope with the derivatives of non-integer order equation. Recently, researchers realized that the fractional calculus could be successfully employed to investigate the neural networks, which named as fractional-order system [16–18]. It should be noted that, the dynamic analysis of fractional-order system is a very promising research topic, and thus attracted worldwide attention [19–24]. Among which, [23], [24] take note of the stability analysis of fractional order memristive one.

Inspired by the above lines, the main task presented below is to learn the state estimation issues of fractional-order memristive model. The main merits of this paper are listed as follows: (i) The memristive model considered in this paper is described by a fractional-order equation instead of integral derivative. Based on the definition of integral derivative, one can see that the integral derivative of a function is only related to its nearby points, while the fractional one has relationship to all of the function history information. As a result, a model described by fractional-order equations possess memory; (ii) Considering the switching character of the memristive system, the target model is translated into a model with uncertain terms, thus the derived conclusions can also be employed to deal with the uncertain system; (iii) The uncertainties that raised in the designation of the observer is also considered.

The remaining sections are structured as: Section 2 is the preliminary knowledge, the main conclusions are voiced in Section 3, simulations figures are illustrated in Section 4. Section 5 provides the conclusion.

2. Preliminaries

In this section, some basic knowledge of Caputo derivative, model description as well as useful Lemmas will be presented.

Definition 2.1. For $\omega(t)$, $D_{t_0,t}^{-\alpha}$ is presented as:

$$D_{t_0,t}^{-\alpha}\omega(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}\omega(\tau)d\tau,$$

where $\Gamma(\cdot)$ is gamma function, fractional order is defined by $\alpha \in \mathbb{R}^+$.

Definition 2.2 [32]**.** The Caputo derivative of $\nu(t)$ implies:

$${}_{C}D_{t_0,t}^{\alpha}\nu(t) = D_{t_0,t}^{-(n-\alpha)}\frac{d^n}{dt^n}\nu(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}\nu^{(n)}(\tau)d\tau,$$

where $n-1<\alpha<n\in Z^+$.

In the following lines, D^α is the abbreviation of ${}_{C}D_{t_0,t}^{\alpha}$.

In this paper, the researchable model is emerge as:

$$D^\alpha x_i(t) = -h_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))g_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t))g_j(x_j(t-\tau)) + L_i, \tag{1}$$

$i=1,\ldots,n,$ where $x_i(t)$ stands for the neuron state, $h_i>0$ implies the connection weights, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t)))$ purport the feedback weights, L_i means the system input. Moreover, $g(\cdot)$ is the activation functions that subjected by:

(A_1) For $\forall s_1, s_2 \in \mathbb{R}$, $g_i(\cdot)$ satisfies:

$$\kappa_i^- \leq \frac{g_i(s_1)-g_i(s_2)}{s_1-s_2} \leq \kappa_i^+, \quad |g_i(\cdot)| \leq M_i.$$

As one knows, in the memristive system, the connection weights will be switched according to the states, i.e., the state variables appeared in (1) can be voiced as:

$$\vartheta(\nu(t)) = \begin{cases} \acute{\nu}, & |\nu| \leq \chi_i, \\ \grave{\nu}, & |\nu| > \chi_i, \end{cases} \tag{2}$$

which implies that $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ switching between $\acute{a}_{ij}$, $\grave{a}_{ij}$, and $\acute{b}_{ij}$, $\grave{b}_{ij}$, respectively.

Let $\overline{a}_{ij}=\max\{\acute{a}_{ij},\grave{a}_{ij}\}$, $\underline{a}_{ij}=\min\{\acute{a}_{ij},\grave{a}_{ij}\}$, $\hat{a}_{ij}=\frac{1}{2}(\overline{a}_{ij}+\underline{a}_{ij})$, $\check{a}_{ij}=\frac{1}{2}(\overline{a}_{ij}-\underline{a}_{ij})$, $\overline{b}_{ij}=\max\{\acute{b}_{ij},\grave{b}_{ij}\}$, $\underline{b}_{ij}=\min\{\acute{b}_{ij},\grave{b}_{ij}\}$, $\hat{b}_{ij}=\frac{1}{2}(\overline{b}_{ij}+\underline{b}_{ij})$, $\check{b}_{ij}=\frac{1}{2}(\overline{b}_{ij}-\underline{b}_{ij})$. Then, system (1) can be modified as:

$$D^\alpha x_i(t) = -h_i x_i(t) + \sum_{j=1}^{n}(\hat{a}_{ij}+\sqcap_{ij}(x_i(t)))g_j(x_j(t)) + \sum_{j=1}^{n}(\hat{b}_{ij}+\sqcup_{ij}(x_i(t)))g_j(x_j(t-\tau)) + L_i, \tag{3}$$