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# Evolution of cooperation on independent networks: The influence of asymmetric information sharing updating mechanism

Jinzhuo Liu[a,b], Haoran Meng[a,b], Wei Wang[a,b,*], Zhongwen Xie[a,b], Qian Yu[a,b]

[a] School of Software, Yunnan University, Kunming, Yunnan 650091, China
[b] Key Laboratory in Software Engineering of Yunnan Province, Kunming 650091, China

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ABSTRACT

In realistic social systems, there exists multiple networks which are influenced by each other with different level of impacts. Inspired by this, we thus study the evolution of public cooperation on two interdependent networks that are connected by means of an asymmetric information sharing updating function, which can construct the interdependent networks in two different classes in order to simulate asymmetric influence of multiple networks which exist in from bacteria to animals as well as human societies. Interestingly, we find that interdependence by means of asymmetric information sharing function can dramatically promote the evolution of cooperation by restraining negative feed-back effect to provide a better environment for cooperators to mushroom. The result shows that the stronger the level of asymmetry, the higher the level of cooperation. And we further inquiry why this asymmetric information sharing updating mechanism can strikingly promote the cooperation in the upper network and find that the value of combined Fermi–Dirac Function of defector in the upper network is relatively larger than the original Fermi–Dirac Function, which can eventually lead to the fraction of cooperation to one in the upper network. Our work can reveal some essential principles in asymmetric interdependent networks that ubiquitously exists in biological and human social systems.



## 1. Introduction

The emergence of cooperation is an omnipresent phenomenon existing ranging from bacteria to animals as well as social systems [1–8]. It remains to be a fascinating problem to search for mechanisms that can generate and maintain the cooperation among the egotism individuals. Prisoner dilemma game, snowdrift game and stag-hunt game have been used to study the cooperation between pairwise interactions and attracted a lot of attention. In particular, the prisoner dilemma game (PDG), served as a paradigm for expressing a social poverty in the case of pairwise interactions, has been used frequently to study such an overarching issue in both theoretical and experimental literature [9–11]. In typical prisoner dilemma game, two players are asked to choose to cooperate or defect simultaneously. They receive a payoff $R$ (reward) if they both choose to cooperate and conversely and they receive a payoff $P$ (punishment) if they both choose to defect. When one player chooses to cooperate while the other chooses to defect, the cooperator receive a payoff $S$ (sucker's payoff) while the

* Corresponding author at: School of Software, Yunnan University, Kunming, Yunnan 650091, China.
*E-mail address:* wangwei@ynu.edu.cn (W. Wang).

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defector receive a payoff $T$ (temptation). The ranking of those payoffs is ordered as $T>P>R>S$, which causes the dilemma that defection is the best strategy for selfish individuals regardless of whatever the opponent's strategy is, but mutual cooperation is best for two prisoners.

Evolutionary game theory which provides a mighty tool to study the biology systems and human society is one of the most important branches in complex network science [12–31]. In the past five years, researchers focused more on interdependent or multiplex networks [32–36] which can describe the interaction between different networks in the real world than on traditional individual network which is isolated and assumed that player interact with direct neighbors on a single network. In reality, our world consists of many different networks and obviously they can be influenced by each other in some ways even if there is no physical links between them. As a result, the real-world networks should be treated as interdependent networks in order to be close to reality. Even small and seemingly irrelevant changes in one network can lead to tremendous impact in other networks, which indicates that the research of the property and function of interdependence between networks is as important as the study of the structures of individual network. One example is the cascade of failures of power network and Internet network that were implicated in an electronic blackout in Italy in September 2003 [37].

In previous work, several mechanisms have been demonstrated to promote cooperation in evolutionary conundrum. Nowak reviewed five rules for the promotion of cooperation named kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection in 2006 [38]. In particular, network reciprocity, is a well-known dynamical rule which can foster the prevalence of cooperation, has inspired many works to investigate the evolution of cooperation on networks including small-world network [39–40], scale-free network [41], interdependent network [42]. Particularly, a seminal research that launched a spree of research aimed at understanding the evolution of cooperation in structured populations was the introduction of spatial structure by Nowak and May [43], which enabled aggregation of cooperators to form compact clusters on the structured network to protect the interior from being exploited by defectors even in the realm of the most challenging prisoner's dilemma game.

In previous research concerning evolutionary game on interdependent networks, they construct two interdependent networks via the utility function [44] in which the interaction between two networks is always symmetric to simulate the realistic situation in which our actions are not motivated solely by our own wellbeing but may also depend on the impact from the others. However, in human society and biology systems, the influence between interdependent networks is mainly not symmetric [45–46]. For examples, in human communication networks, a famous person can interact directly with another famous person and the action they took may violently influence the action in the network of lower class consisting of a large number of fans. As a result, it is of great interest to explore the mechanism of asymmetry in evolutionary game theory.

In this paper, we study the evolution of prisoner's dilemma game on two interdependent networks, which are linked together via an asymmetric information sharing updating function that is defined as combination of individuals' Fermi function of related pairs of players. In this way, players in the upper network consider the condition of the opposite player. And the reason why we choose Fermi–Dirac Function as the bridge to connect two networks is that Fermi–Dirac Function contains a lot of information including all the interactions between the player and its four nearest neighbors so that it can better describe the condition of the player. Excitingly, we find that the interdependent by means of asymmetric information sharing updating function can dramatically promote the cooperation in the interdependent networks system especially that in the upper network. The result arouses our interest and we define "the fraction of defectors who are more inclined to cooperate" as $f_{D\to C}$ and similarly, we also define the "the fraction of cooperators who are more inclined to defect" as $f_{C\to D}$. We compute $f_{D\to C}$ and $f_{C\to D}$ in every Monte Carlo steps to explore why asymmetric information sharing updating mechanism can highly promote the cooperation in the upper network. From the distribution of $f_{D\to C}$ and $f_{C\to D}$, we find that about sixty to seventy percent of defectors become more intended to change their strategies than the defectors in the situation where they locate at isolated network without asymmetric information sharing updating mechanism and cooperators become more conservative comparing to the cooperators in traditional isolated networks. Thus, we conclude that asymmetric information sharing updating mechanism is the most significant cause for promoting cooperation.

## 2. Model

Here, we consider the weak prisoner's dilemma game with normalized payoff and the payoff matrix is defined as:

$$\mathrm{A} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{1}$$

where the parameter $b$ denotes the temptation to defect. And $b$ is the main parameter in prisoner dilemma game.

The prison dilemma game on both networks is staged on a $L \times L$ square lattice with period boundary condition, where cooperators and defectors are randomly populated with equal probability, which can be described as:

$$S_C = (1, 0)^T,\ S_D = (0, 1)^T. \tag{2}$$

The evolution of the initial strategy distribution is performed by repeating the following elementary steps in accordance with Monte Carlo simulation procedure. And the Mote Carlo simulation steps are explained as follow. First of all, we have the probability of fifty percent to update the strategy of a random chosen player in the upper network first and then the